SCIENTIFIC Publishing Limited

Journal of Intelligent Communication
http://ojs.ukscip.com/index.php/jic

Article

The Swedish Production of Apartments as a Function of GNP, Building Costs and Population Changes: Generation of Intelligent Media Content via Big Data Analytics

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Received: 11 May 2024; **Revised**: 14 June 2024; **Revised**: 14 June 2024; **Accepted**: 18 June 2024; **Published**: 29 June 2024

Abstract: The production of new apartments in Sweden has varied strongly during the period from 1975 to 2021. A new statistical function, which explains these production changes, has been developed. This function is designed, based on a set of hypotheses of how the production level should be affected by different explaining factors, such as the GNP, the size of the population, the growth of the population, and the cost of construction. The following hypotheses could not be rejected: the apartment production is a strictly increasing and strictly convex function of GNP, and a strictly increasing function of the size of the population and the growth of the population, and a strictly decreasing function of the cost of construction. The parameters of the statistical function have been estimated with high precision, via multiple regression analysis. It was not possible to detect heteroscedasticity via residual analysis. Furthermore, no indications that nonlinear transformations would improve the selected model were found. The apartment production model contains a strongly significant negative time trend. The estimated function is used to predict the future apartment production until the year 2050. The predictions are based on assumed growth levels of GNP and the population, and on alternative future time trends of the construction cost index. If the real construction cost index continues to grow with the same average trend as from the year 1993 to 2021, the future apartment construction level will stay almost constant at 40,000 apartments per year until 2050. If the future real construction cost index stays constant at the level in 2022, the production of new apartments will grow to almost 90,000 apartments per year in 2050. If the real construction cost index can be decreased to the level in 1993, the production of new apartments will grow to almost 130,000 apartments per year in the year 2050.

Keywords: construction industry; apartments; statistical analysis; predictions

1. Introduction

The production of houses and apartments in the construction industry represents the investments that play key roles in most societies.

In the countryside of some countries, when a person plans to build a small house to live in, the decisions may be rather simple. The decision to build the house follows individual preferences and known parameters and very few persons are involved. In cities, many firms, authorities and interest groups are often involved in the building process. Several decisions must be coordinated in time and space, which complicates the planning.

2. Literature Review

If we are interested in understanding and improving the dynamics of the construction industry, and in particular the investments in new residential buildings, it is a natural first step to consult the area of fundamental economic optimization. Gat (1995) [1] argues for economic optimization approaches to build projects that simultaneously consider multiple periods, the speed of construction, relevant cost functions, and the rate of interest in the capital market. Dipasquale (1999) [2], however, states that very little is known about the housing supply. According to the reference, it is very important to increase the understanding of the micro-economic foundations of housing supply. Pyhrr et al. (1999) [3] write that they have found that cycles strongly influence the construction sector. They try to summarize and understand what they call "real estate cycles". They claim that many different cycles affect each other and the real estate market. Witkiewicz (2002) [4] continues with the cycle theories and tries to construct indicators of a Swedish real estate cycle.

From the perspectives of labor market policy, housing policy and construction policy, it is important to understand how fundamental economic variables, such as GDP, are linked to the activities in the construction industry and others. Wigren et al (2007) [5] investigate such relationships. They find indications of some dependencies but claim that the performed Granger causality tests are not conclusive. The GDP growth rate can also be studied as a function of several explaining variables. This is done by Batrancea et al. (2021) [6], who investigated the economic determinants of economic growth in 34 countries across Africa during a two-decade period (2001–2019). Ott et al. (2012) [7] are interested in the dynamics of large residential development projects. They investigate different policies and find that very different production plans, for instance, with continuous and discontinuous construction intervals, can all be optimal under suitable conditions. Hence, it is necessary to know specific details and locally relevant parameters before suggesting a particular strategy.

In order to understand the logic and dynamics of urban development, it is necessary to understand the market forms and the decision processes that govern the construction sector. Guthrie (2022) [8] compares the optimal urban dynamic development decisions that should be made for competitive firms and a monopolist landowner. It is found that competition may lead to a welfare maximum. Furthermore, a monopolist landowner should develop the land faster, but with less building density than in a competitive market. The lower building density implies that the future profits of the monopolist are higher, since it will not be possible for other developers to increase the number of houses in the area above the point where the monopolists' profit is maximized.

Sometimes, in order to understand how and why new buildings appear, it is not only necessary to be aware of fundamental economic conditions and relations. Barr (2012) [9] claims that the construction of skyscrapers and other tall buildings does not only provide the developer with profits. Social status and other benefits may follow. Barr et al (2021) [10], write that political factors and intercity competition are important motives for building tall buildings. They also report that they find such tendencies in China.

Of course, it is not only interesting to understand the supply of housing. The housing demand and related topics, such as the households' dwelling valuations and mortgage decisions, are studied by Koblyakova et al. (2022) [11]. They conclude that biased subjective valuations and housing decisions are inseparable. According to the authors, this fact causes imperfections in the housing market.

In cities in countries such as Sweden, almost all activities in the construction industry are regulated by laws, and local governments must give detailed permissions concerning most properties of new constructions. Furthermore, in Sweden, the rents of apartments are strictly regulated. For these reasons, it is difficult to completely understand and control the planning of the building process based on market economic theories and methods.

In this paper, the ambition is to find the fundamental factors that influence the production level in the building industry in Sweden. Official and detailed statistics will be used to test a set of hypotheses of how these factors affect the level of building activity. A function will be estimated that gives the apartment production level as a function of the explaining factors and a stochastic residual. This will be done using multiple regression analysis, minimizing the variance of the residuals.

Statistics Sweden (2023) [12] has a long tradition of collecting detailed statistical information and making it publicly available via web pages. Figure 1 shows the number of apartments finished in Sweden, $A(t)$, in different years from 1975 to 2021. The year is denoted by t. Clearly, the variation is considerable. In this paper, we abstract from notational complications connected to apartments that are only partly constructed during a particular year. Hence, "finished apartments" are sometimes denoted "produced apartments", even if some apartments' construction may start at the end of the year t and stop at the beginning of the next year $t + 1$.

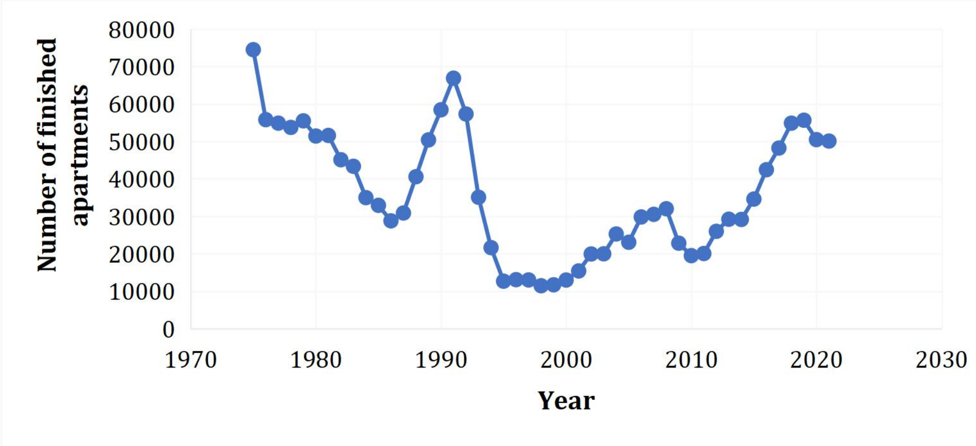

Figure 1. Number of apartments finished in Sweden, $A(t)$, from 1975 to 2021. Source: Statistics Sweden (2023).

The Swedish GNP, also denoted $G(t)$, has grown steadily over time, with two main exceptions, namely the periods from 1990 to 1993 and 2007 to 2009. This is found in Figure 2.

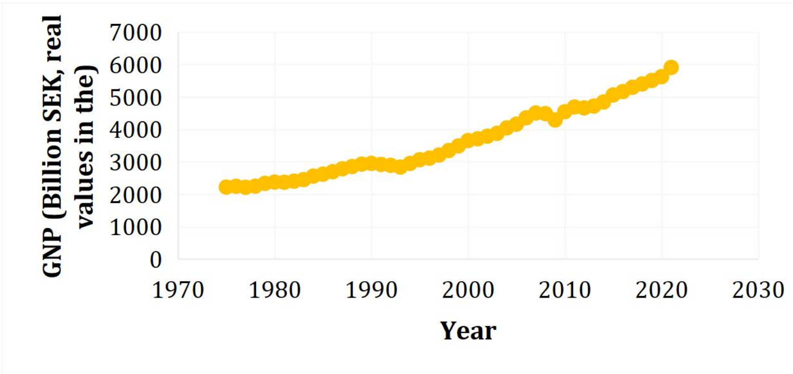

Figure 2. Real Gross National Product in Sweden, $G(t)$, from 1975 to 2021. Unit: Billion SEK, real prices in the level of 2022. Source: Statistics Sweden (2023).

Figure 3 shows that the population growth has considerable variation. The period 2014 to 2019 was strongly influenced by large immigration. The population is shown in Figure 4.

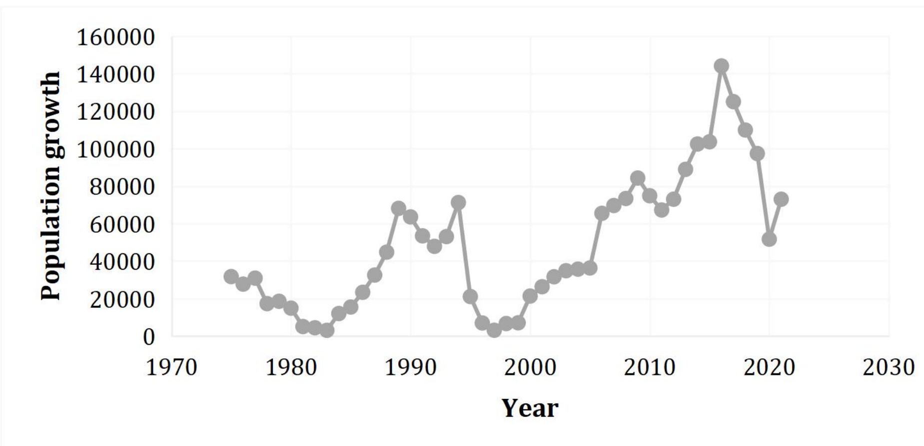

Figure 3. Population growth per year in Sweden, $P(t) - P(t-1)$, from 1975 to 2021. Compare with Figure 4, where $P(t)$ is found. Source: Statistics Sweden (2023).

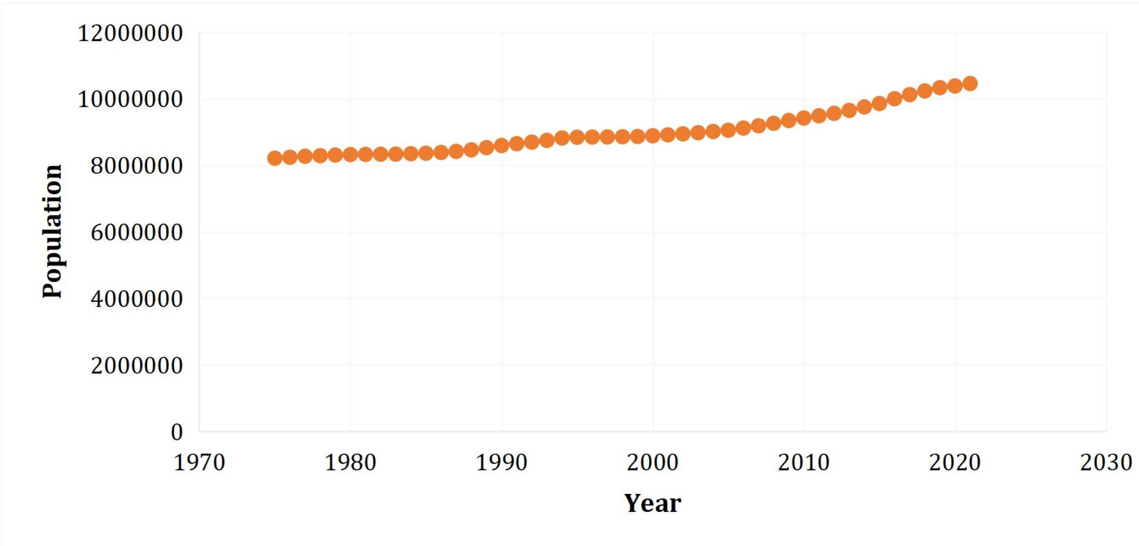

Figure 4. Population in Sweden, $P(t)$, from 1975 to 2021. Source: Statistics Sweden (2023).

As shown in the later part of this paper, the building costs strongly influence the investments in new apartments. Figure 5 shows the time path of the real building costs. From 1975 to 1990, this cost index was almost constant, close to 67. The index fell until 1993–1994 when the real building cost index was 61, and then strongly increased until 2022 when the index reached 100.

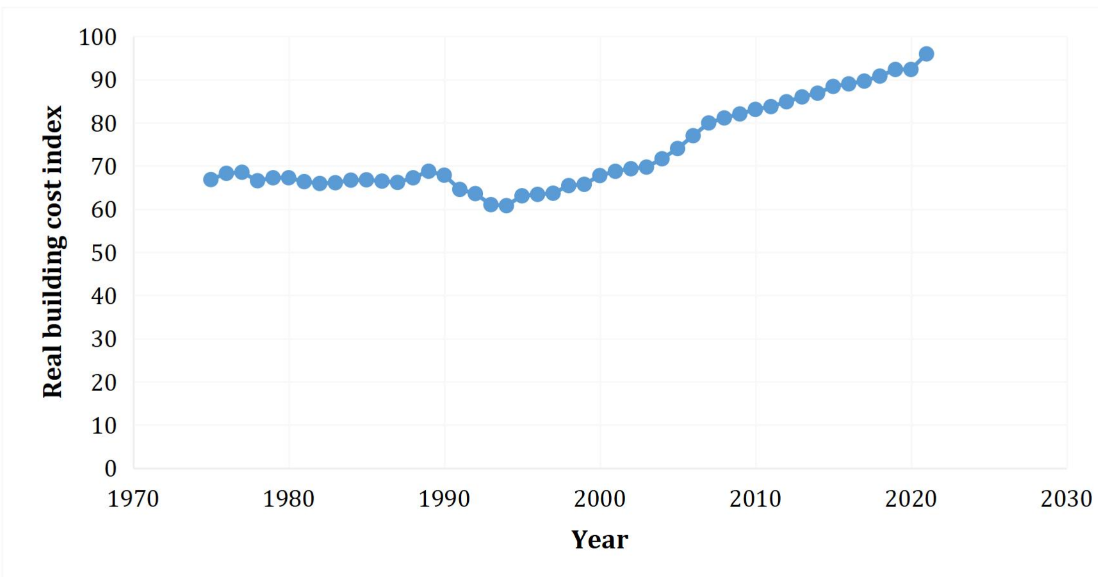

Figure 5. Real building cost index in Sweden, $C(t)$, from 1975 to 2021. The index takes the value 100 in year 2022. Source: Statistics Sweden (2023).

3. Materials and Methods

First, official and publicly available empirical data from Statistics Sweden, covering the building sector, economics, and the population, is used to explain the historical levels of apartment production. Then, the dependent predictions of building cost will be made until 2050. In the Appendix, the empirical data is included.

Furthermore, the statistical properties of the empirical data are described in Tables B1 (Descriptive Statistics) and B2 (Correlation Matrix). The statistical analysis is performed with Excel.

3.1. General Ideas and Motivation for the Hypotheses

First, we will define some hypotheses concerning how fundamental factors should influence the production level in the building industry. When a model is defined, it is important to consider and motivate the functional form. We assume that the decision-makers who determine if new apartments should be built are interested in maximizing the utility of the people who should live in the area. Figure 6 illustrates a possible shape of the function of the utility of living, as a function of the living space *S*. Note that the utility is assumed to be a strictly increasing function of *S*. The function is strictly concave, which means that the marginal utility of living space is a decreasing function of *S*. Furthermore, the utility function has a strictly positive third derivative. The slope of the function gradually decreases, but always stays strictly positive. These properties are reasonable when we study the utility of living space *S*. The function, and more generally, the functional form, illustrated in Figure 6, are logarithmic.

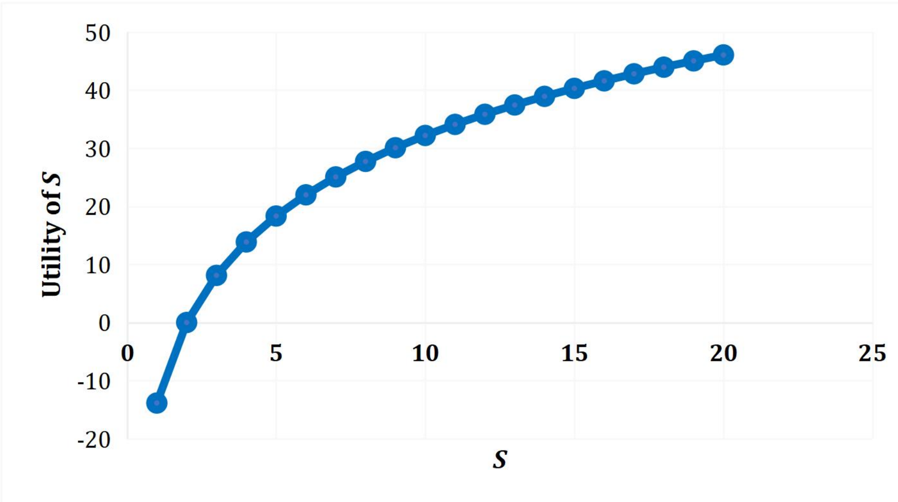

Figure 6. Assumed functional form of the utility of *S* as a function of *S*. The parameters are found in Table 1.

Table 1. A computer code with a numerical specification of the optimization problem. The code also derives the explicit optimal analytical solutions.

```
max = U;
U = f1*F - f2/2*F^2 + s0*@LOG(s1*S);
[budget] pf*F + ps*S < B;
f1 = 20; f2 = 1; s0 = 20; s1 = 0.5; pf = 1; ps = 1; B = 10;

S_opt = s0/(f1-f2*F_opt)*(pf/ps);
F_not_feasible = f1/(2*f2) + B/(2*pf) + ((f1/(2*f2) + B/(2*pf))^2  + s0/f2 - B*f1/(pf*f2) ) ^0.5;
F_opt = f1/(2*f2) + B/(2*pf) - ((f1/(2*f2) + B/(2*pf))^2  +  s0/f2 - B*f1/(pf*f2) ) ^0.5;
U_opt = f1*F_opt - f2/2*(F_opt)^2 + s0*@LOG(s1*S_opt);
Lambda_opt = s0/(ps*S_opt);
end
```

In Figure 7, we find the assumed utility of *F*, as a function of *F*. *F* represents "fundamental investments", except for living space, *S*, which also must be considered when the use of the total budget for investments should be distributed. The utility of *F* is strictly increasing for low levels of *F*. The slope of the function decreases as a linear function of *F*. Hence, *F* has a unique maximum. The utility function gives a reasonable representation of the utility of several things, such as the capacity of infrastructure, the size of the police force, etc. If we invest too much, the net utility may even be negative.

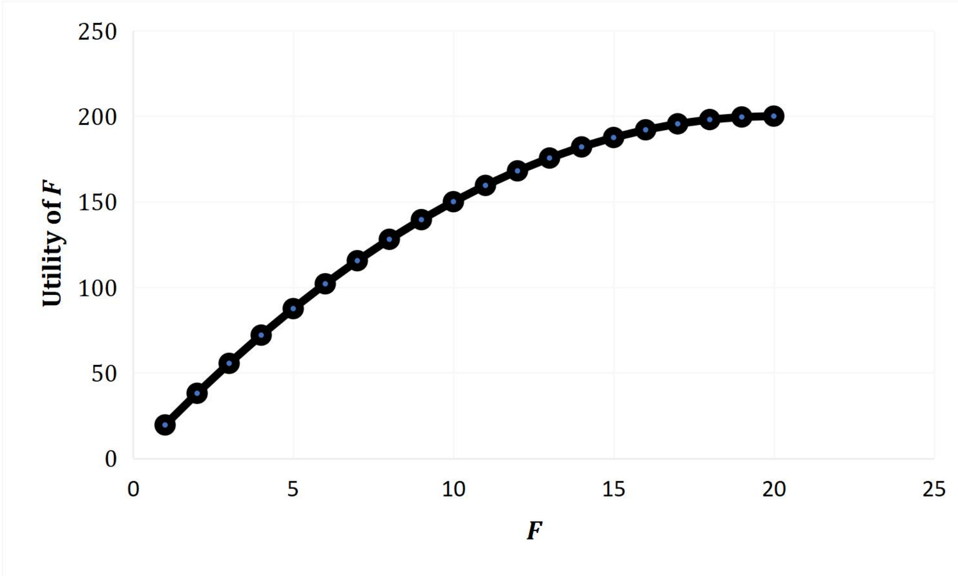

Figure 7. Assumed functional form of the utility of F as a function of F. The parameters are found in Table 1.

In the general utility optimization problem, we should simultaneously consider investments in living space, S, other fundamental investments, F, and the total budget, B. In Figure 8, we find a three-dimensional illustration of the utility function, defined as the sum of the two functions found in Figures 6 and 7. In Equation (1), this utility function is maximized, via the decision variables F and S. f_1, f_2, s_0, and s_1 are strictly positive parameters. The maximization is performed subject to the budget constraint Equation (2). The prices per unit of F and S are p_F and p_S respectively. In Equation (3), we find the Lagrange function of the constrained optimization problem.

In the optimization, we only consider interior solutions, where $S > 0$, $F > 0$ and the marginal value of B, the "shadow price of the budget", λ, are strictly positive. Then, according to the Karush-Kuhn-Tucker-conditions, the three simultaneous equations in Equation (4), must be satisfied by the optimal solution. In the later part of the analysis, we will find that the optimal values of S, F and λ are really strictly positive, when they are determined from the equations in Equation (4), which confirms that the initial assumptions were correct.

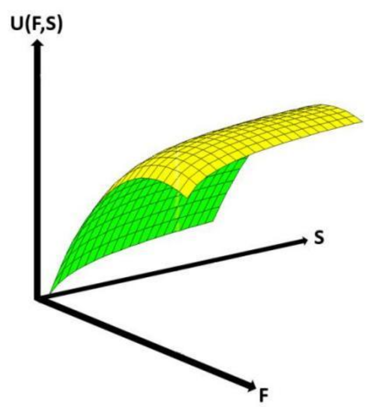

Figure 8. The utility, U, of F and S, as a function of F and S. The parameters are found in Table 1.

$$\max_{F,S} U = f_1 F - \frac{f_2}{2} F^2 + s_0 LN(s_1 S) \tag{1}$$

s.t.

$$p_F F + p_S S \leq B \tag{2}$$

$$L = f_1 F - \frac{f_2}{2} F^2 + s_0 LN(s_1 S) + \lambda\left(B - p_F F - p_S S\right) \tag{3}$$

$$\begin{cases} \dfrac{dL}{dF} = \quad f_1 - f_2 F - p_F \lambda \quad = 0 & (4a) \\ \dfrac{dL}{dS} = \quad s_0 S^{-1} - p_S \lambda \quad = 0 & (4b) \\ \dfrac{dL}{d\lambda} = \quad -p_F F - p_S S + B \quad = 0 & (4c) \end{cases} \tag{4}$$

From Equation (4a) we get Equation (5) and from Equation (4b), we obtain Equation (6). Clearly, the optimal marginal value of the budget, λ, is the same, if it is used to produce *F* and *S*. The optimal marginal value of the budget equals the derivative of the utility of *F* (or *S*), divided by the price per unit of *F* (or *S*), as shown in Equation (5) and Equation (6).

$$\lambda = \frac{f_1 - f_2 F}{p_F} \tag{5}$$

$$\lambda = \frac{s_0 S^{-1}}{p_S} \tag{6}$$

From Equations (5) and (6), we get Equation (7). There, we see that the optimal level of *S* is a product. The first factor is a ratio: s_0 divided by the marginal utility of *F*. The second factor is also a ratio, namely a relative price: the price of *F* divided by the price of *S*. Hence, the optimal value of *S* is a strictly decreasing function of the price of *S* and a strictly increasing function of s_0. *S* is a strictly increasing function of the price of *F* and a strictly decreasing function of the marginal utility of *F*.

$$S = \frac{s_0}{\left(f_1 - f_2 F\right)} \times \frac{p_F}{p_S} \tag{7}$$

From Equations (4c) and (7), we get Equation (8). In Equation (8), we have replaced *S* by the optimal value of *S*.

$$p_F F + p_S\left(\frac{s_0}{\left(f_1 - f_2 F\right)} \times \frac{p_F}{p_S}\right) - B = 0 \tag{8}$$

The optimal value of *F* can now be determined as a function of the parameters. Equation (8) can be simplified to Equations (9) and (10).

$$F + \frac{s_0}{\left(f_1 - f_2 F\right)} - \frac{B}{p_F} = 0 \tag{9}$$

$$\left(f_1 - f_2 F\right) F + s_0 - \frac{B}{p_F}\left(f_1 - f_2 F\right) = 0 \tag{10}$$

Equation (10) is via Equations (11) and (12) reformulated to Equation (13), a quadratic equation in standard form, that can be used to determine the optimal value of F from an explicit function found in Equation (14).

$$f_1 F - f_2 F^2 + s_0 - \frac{B f_1}{p_F} + \frac{B f_2}{p_F} F = 0 \tag{11}$$

$$-f_2F^2+\left(f_1+\frac{Bf_2}{p_F}\right)F+\left(s_0-\frac{Bf_1}{p_F}\right)=0 \tag{12}$$

$$F^2-\left(\frac{f_1}{f_2}+\frac{B}{p_F}\right)F+\left(\frac{Bf_1}{p_Ff_2}-\frac{s_0}{f_2}\right)=0 \tag{13}$$

$$F=\frac{f_1}{2f_2}+\frac{B}{2p_F}\begin{matrix}(+)\\-\end{matrix}\sqrt{\left(\frac{f_1}{2f_2}+\frac{B}{2p_F}\right)^2+\frac{s_0}{f_2}-\frac{Bf_1}{p_Ff_2}} \tag{14}$$

When we apply Equation (14), two different real solutions of F, denoted F_1 and F_2, are found. F_1 is the solution when the negative sign is selected before the square root expression in Equation (14) and F_2 is obtained if the positive sign is used. One of these F values, F_1, will satisfy the budget constraint found in Equations (2) and (4c). F_1 contributes to the maximization of the utility function. F_2 does not satisfy the budget constraint and can be shown to lead to an unfeasible minimum. The optimization problem expressed in Equations (1) and (2) contains a strictly concave objective function and a linear constraint. In such a problem, if the first-order optimum conditions are satisfied and the solution is feasible, the solution is a unique and feasible maximum.

The optimal value of S can now be found via $F = F_1$, the parameters, and Equation (7). Then, the optimal value of λ can be derived via Equations (5) or (6). Finally, the optimal value of U is found via Equation (1), the parameters, F_1 and S.

3.2. Comparison of the Analytical Results with Results from Numerical Optimization

In the first four rows of Table 1, an optimization code in the modeling language Lingo is found, which solves a numerically specified version of the problem defined in Equations (1) and (2). At the end of the program code, the optimal values of the variables are calculated via the analytical method developed in Equations (1)–(14). In Table 2 we see the results from the program. The optimal solutions from the numerical calculations and the analytical derivations are equal, which confirms that the results are correct. The numerical version of the solution is more useful if other functional forms are used in the utility function, and/or if the analysis is expanded to a larger number of possible investments. In such cases, explicit optimal solutions are usually not possible to derive analytically.

Table 2. Results from the computer code with a numerical specification of the optimization problem.

Variable	Value	Reduced Cost
U	128.3049	0.000000
F1	20.00000	0.000000
F	8.291796	0.000000
F2	1.000000	0.000000
S0	20.00000	0.000000
S1	0.5000000	0.000000
S	1.708204	0.1189181E-06
PF	1.000000	0.000000
PS	1.000000	0.000000
B	10.00000	0.000000
S_OPT	1.708204	0.000000
F_OPT	8.291796	0.000000
F_NOT_FEASIBLE	21.70820	0.000000
U_OPT	128.3049	0.000000
LAMBDA_OPT	11.70820	0.000000

Figure 9 shows how the optimal decisions, F and S, are affected by alternative levels of the budget, B. The optimal level of F is a strictly increasing and strictly concave function of B. We observe that the optimal level of S is a strictly increasing and strictly convex function of B. This is an important observation. This leads to the following hypotheses:

Hypothesis 1. The apartment production is a strictly increasing and strictly convex function of GNP.

If the number of apartments A increases, the living space S will increase. In the statistical analysis, we may consider the gross national product (GNP) to be proportional to the budget B. In case the investment process really maximizes the utility of the people in the country, the optimal production of apartments A should be a strictly increasing and strictly convex function of the GNP. The exact and relevant level of convexity of the relationship between the apartment construction level and the GNP level is not known. We assume that the GNP should be raised to 1.5 to reflect how it influences the apartment construction level.

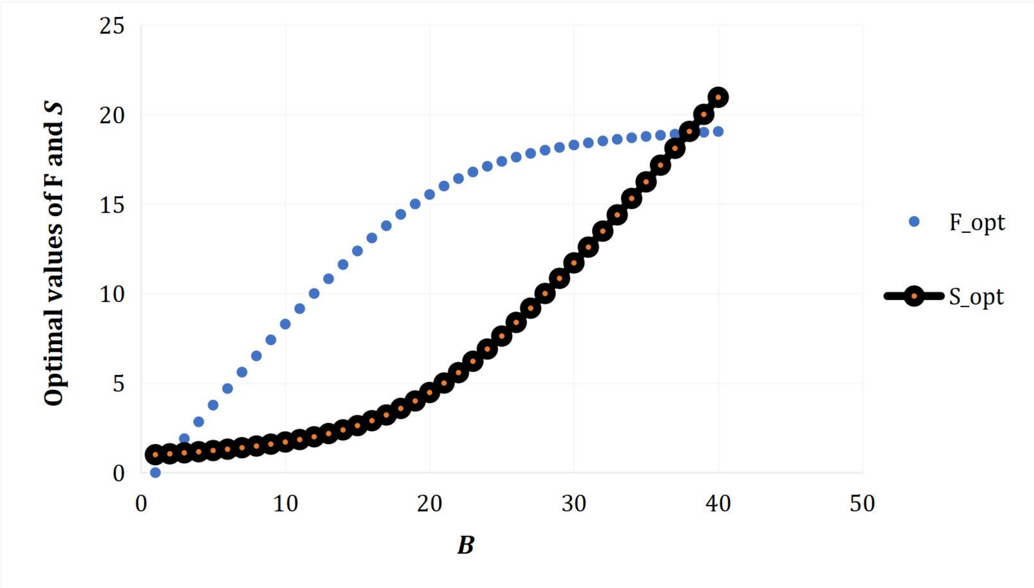

Figure 9. Optimal decisions for alternative levels of the budget B. The optimization problem is defined in Equations (1) and (2). All parameters, except for B, have the numerical values defined in the computer code in the numerical example, in Table 1.

In Figure 10, We observe that the marginal capacity value, λ, is a strictly decreasing and strictly convex function of B. Figure 11 shows that the optimal objective function value is a strictly increasing and strictly concave function of B.

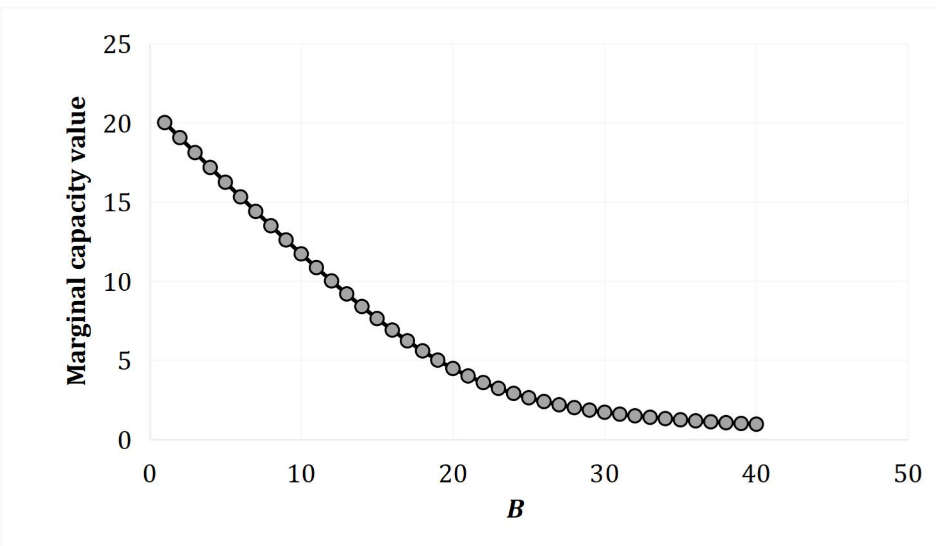

Figure 10. The marginal capacity value of the budget, λ, Lambda, for alternative levels of the budget B. The optimization problem is defined in Equations (1) and (2). All parameters, except for B, have the numerical values defined in the computer code in the numerical example, in Table 1.

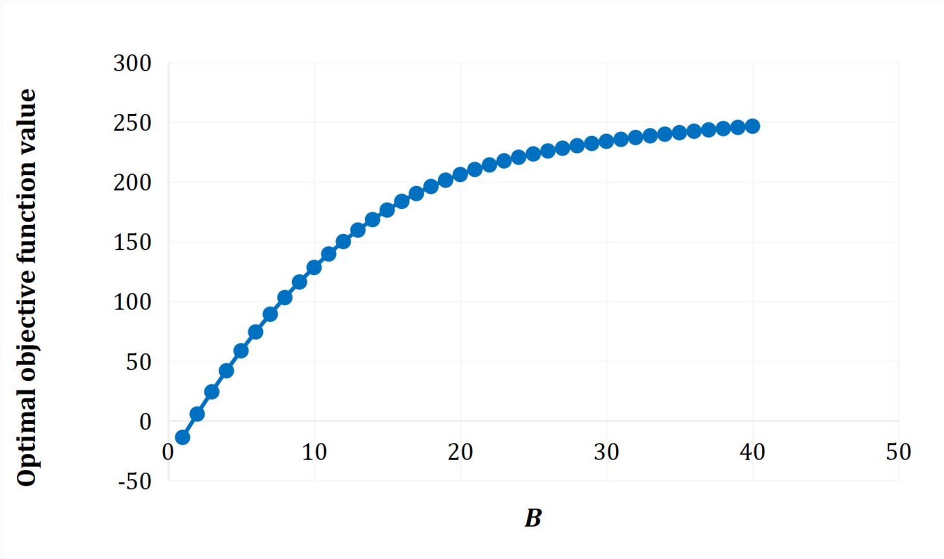

Figure 11. The optimal objective function value, U, for alternative levels of the budget B. The optimization problem is defined in Equations (1) and (2). All parameters, except for B, have the numerical values defined in the computer code in the numerical example, in Table 1. We observe that the optimal objective function value is a strictly increasing and strictly concave function of B.

We may form Hypotheses 2–4:

Hypothesis 2. The apartment production is a strictly increasing function of the size of the population.

It is reasonable that the living space and the number of apartments should be increasing functions of the number of individuals in the population.

Hypothesis 3. The apartment production is a strictly increasing function of the growth of the population.

If the population increases slowly, it is not very difficult to gradually adjust the use of existing apartments and houses, which means that the number of new apartments does not have to increase very much. If the population increases rapidly, it is more urgent to instantly build new apartments, where the new individuals can live. In countries without rent regulations, with a rapidly increasing population, the prices of all apartments will usually increase rapidly. Individuals in the initially existing population may sell their old apartments to the new inhabitants and afford to move to modern new apartments.

Hypothesis 4. The apartment production is a strictly decreasing function of the cost of construction.

During the latest decades, the costs of apartment production in Sweden have increased with time. This may partly be caused by more competition for limited and expensive space in large cities. Furthermore, in Sweden, the expanding building regulations have become more complicated to satisfy. It takes time and money to adapt to new rules and to negotiate with different authorities and interest groups that can stop and influence a building project. These things imply that the building costs increase with time. Likely, the apartment production level is negatively affected by increasing construction costs per apartment. The hypotheses are presented in compact form in Table 3.

Table 3. Hypotheses.

Index	Hypothesis
H1	The apartment production is a strictly increasing function of $(\text{GNP})^{1.5}$.
H2	The apartment production is a strictly increasing function of P.
H3	The apartment production is a strictly increasing function of dP/dt.
H4	The apartment production is a strictly decreasing function of C.

Below, multiple regression analysis and official statistics are used to test Hypotheses 1–4. A function is estimated that gives the apartment production level as a function of the explaining factors and a stochastic residual. The estimated function is found in Equation (29), where the parameters are denoted by k_1–k_5. The function contains no intercept since it cannot be logically motivated. The variable definitions are found in Table 4.

$$\hat{A} = k_1 \times G^{1.5} + k_2 \times P + k_3 \times \frac{dP}{dt} + k_4 \times C + k_5 \times (t - 1975) + \varepsilon_t \quad (29)$$

Finally, the estimated function is used to predict the future apartment production until 2050. The predictions are based on assumed growth levels of GNP and the population, and on alternative growth levels of the costs of construction.

Table 4. Variables in the regression analysis.

Variable	Explanation	Unit
t	Time.	Year.
$G(t)$	Gross National Product (GNP).	Billion SEK, real values, in the prices level of 2022.
$P(t)$	Population.	Number of individuals.
$dP/dt(t)$	Population growth per year (approximated from $P(t) - P(t-1)$).	Number of individuals per year.
$C(t)$	Real building cost index.	The index is 100 in 2022.

4. Results

The production of new apartments in Sweden has varied strongly during the period 1975 to 2021. A new statistical function, which explains these production changes, has been developed. This function is designed based on a set of Hypotheses 1–4 of how the production level should be affected by different explaining factors, such as the GNP, size of the population, growth of the population, and cost of construction.

Tables 5–7 contain the regression analysis results. Some of the important results are the following: Table 5 shows that R^2 exceeds 95%; Table 6 reveals that F is almost 180 and that the significance of F is close to 9×10^{-27}; in Table 7, we find that the four Hypotheses 1–4, cannot be rejected. There is a significant negative time trend in the function, represented by the coefficient of the variable "t—1975". The coefficients of $G^{1.5}$, P, dP/dt, and C, all have the expected signs. All P values of these parameters are below 5%. Some of these parameters have much lower P values. The P value of the coefficient of $G^{1.5}$ is less than 6×10^{-6}.

Table 5. Regression statistics part 1.

Regression Statistics	
Multiple R	0.977451718
R^2	0.955411861
Adjusted R^2	0.927355848
Standard Error	8874.547727
Observations	47

Table 6. Regression statistics part 2.

ANOVA					
	df	***SS***	***MS***	***F***	***Significance F***
Regression	5	70878257812	14175651562	179.9909093	9.0939E-27
Residual	42	3307819089	78757597.36		
Total	47	74186076901			

Table 7. Regression statistics part 3.

	Coefficients	Standard Error	*t* Stat	*P*-value
Intercept	0	#N/A	#N/A	#N/A
$G^{1.5}$	0. 576758329	5.200156572	5.200156572	5.54572E-06
P	0.010316769	2.839195213	2.839195213	0.00694128
C	−1233.47247	−2.12744601 5	−2.12744601 5	0.039297794
t-1975	−4810.590291	−7.335975053	−7.335975053	4.82588E-09
dP/dt	0.302168062	4.966393914	4.966393914	1.1872E-05

The estimated value of A is:

$$\hat{A} = 0.57676 \times G^{1.5} + 0.010317 \times P + 0.30217 \times \frac{dP}{dt} - 1233.5 \times C - 4810.6 \times (t - 1975) + \varepsilon_t \tag{30}$$

These hypotheses could not be rejected: the apartment production is a strictly increasing and strictly convex function of GNP, and a strictly increasing function of the size of the population, and of the growth of the population. The apartment production is a strictly decreasing function of the cost of construction. The apartment production also contains a strongly significant negative time trend.

The parameters of the regression function have been estimated with high precision, via multiple regression analysis. In the Appendix, residual diagrams A1–A6 are included. It is not possible to detect heteroscedasticity via the residual analysis. Furthermore, no indications that nonlinear transformations would be able to improve the selected model are found.

Figures 12–16 show the true values and the predicted values of the number of produced apartments, A, for the alternative values of the explaining variables. The model results are convincing. The graphs show that the model predictions reproduce the true historical values with small random errors, in all variable dimensions.

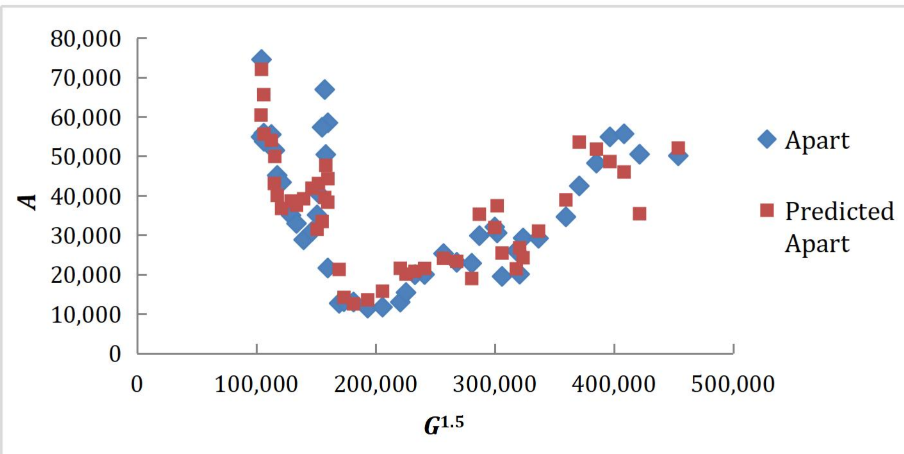

Figure 12. The number of produced apartments, A, from 1975 to 2021: true values (Apart, blue) and predictions (Predicted Apart, red) according to the estimated function, for different values of the factor $G^{1.5}$, the GNP raised to 1.5.

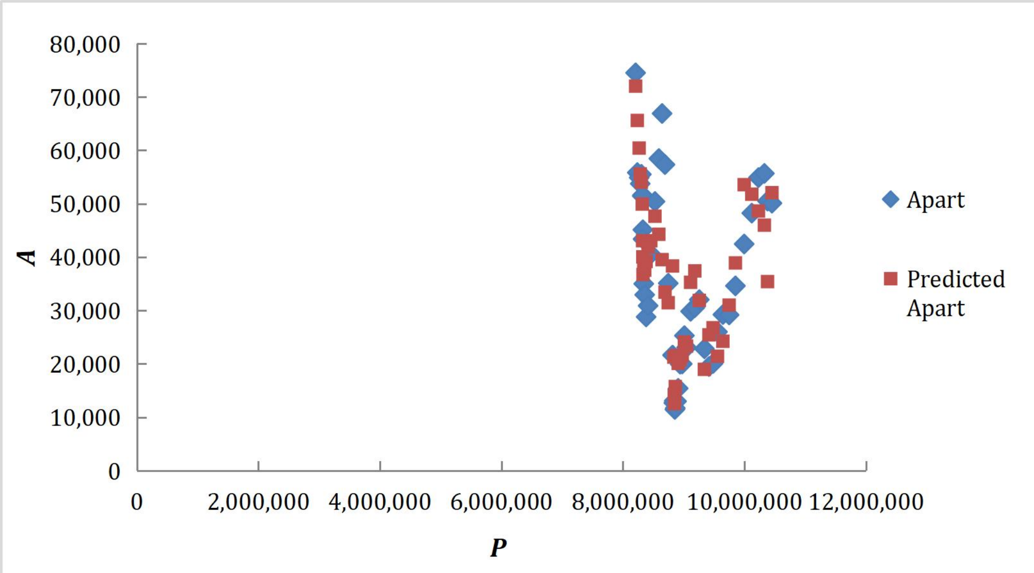

Figure 13. The number of produced apartments, A, from 1975 to 2021: true values (Apart, blue) and predictions (Predicted Apart, red) according to the estimated function, for different values of the factor P, the size of the population.

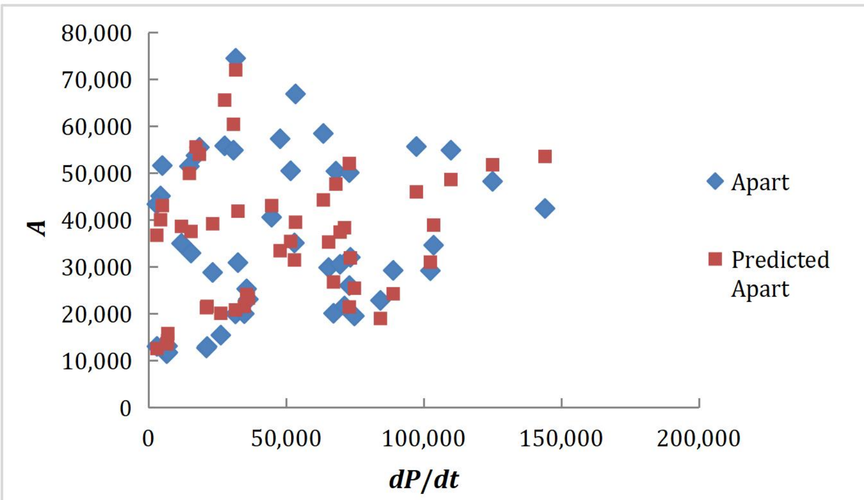

Figure 14. The number of produced apartments, A, from 1975 to 2021: true values (Apart, blue) and predictions (Predicted Apart, red) according to the estimated function, for different values of the factor dP/dt, an approximation of $P(t) - P(t-1)$, the growth of the population.

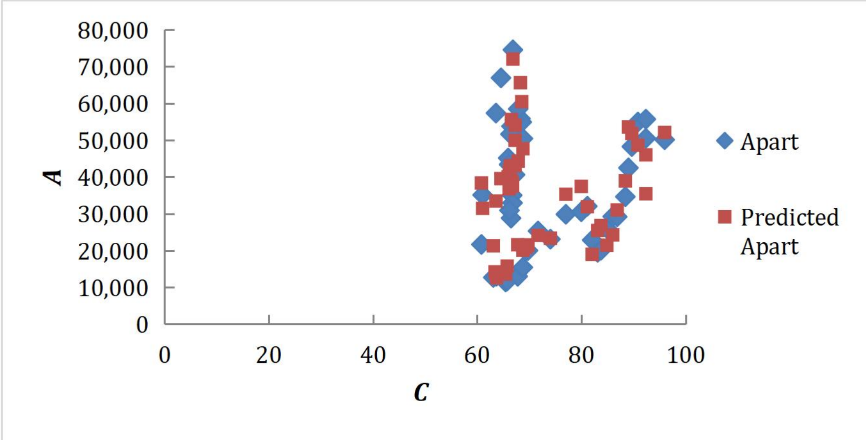

Figure 15. The number of produced apartments, *A*, from 1975 to 2021: true values (Apart, blue) and predictions (Predicted Apart, red) according to the estimated function, for different values of the factor *C*, the real building cost index.

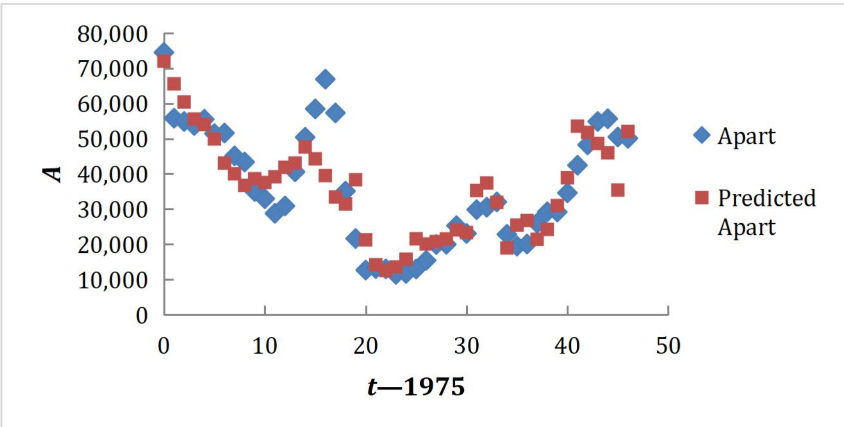

Figure 16. The number of produced apartments, *A*, from 1975 to 2021: true values (Apart, blue) and predictions (Predicted Apart, red) according to the estimated function, for different values of the factor *t*—1975.

4.1 Predictions into the Future

The estimated function is used to predict the future apartment production until 2050. The predictions are based on assumed growth levels of GNP and the population, and on alternative levels of the cost of construction. In Figure 17, we find the GNP prediction, based on the assumption that the derivative of GNP with respect to time, will be the same as the average value of the derivative from 1975 to 2021. In Figure 18, the population prediction is illustrated. The hypothesis is that the derivative of *P* with respect to time will be the same as the average value of the derivative from 1975 to 2021.

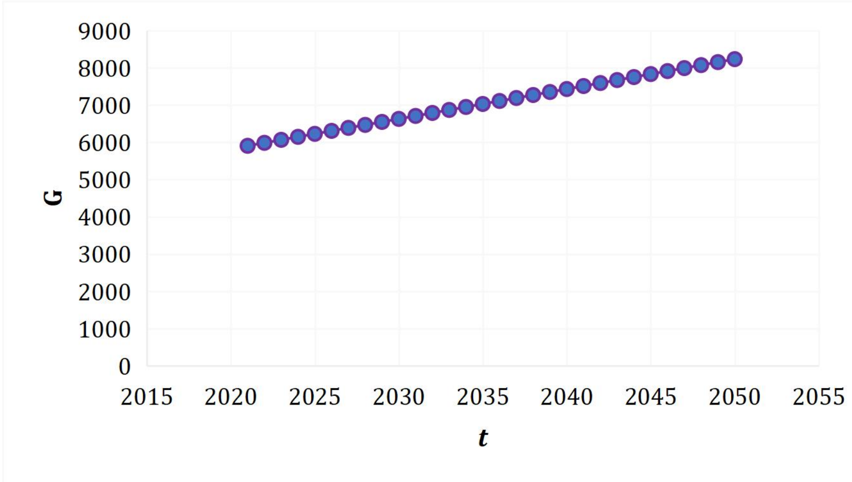

Figure 17. The future time path of GNP, G, from year 2021 to 2050, under the assumption that the derivative of GNP with respect to time will be the same as the average value of the derivative from 1975 to 2021.

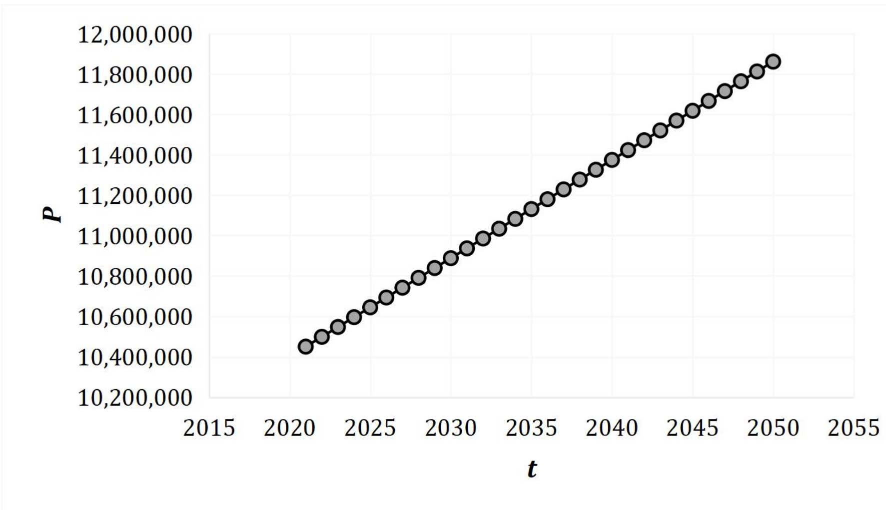

Figure 18. The future time path of the population, P, from 2021 to 2050, under the assumption that the derivative of P with respect to time will be the same as the average value of the derivative from 1975 to 2021.

The future time paths of C from 2021 to 2050 are illustrated in Figure 19 under four alternative assumptions. The details of these assumptions are reported in connection to Figure 19. The future time paths of A from 2021 to 2050 are found in Figure 20 under the four alternative assumptions that are illustrated in Figure 19.

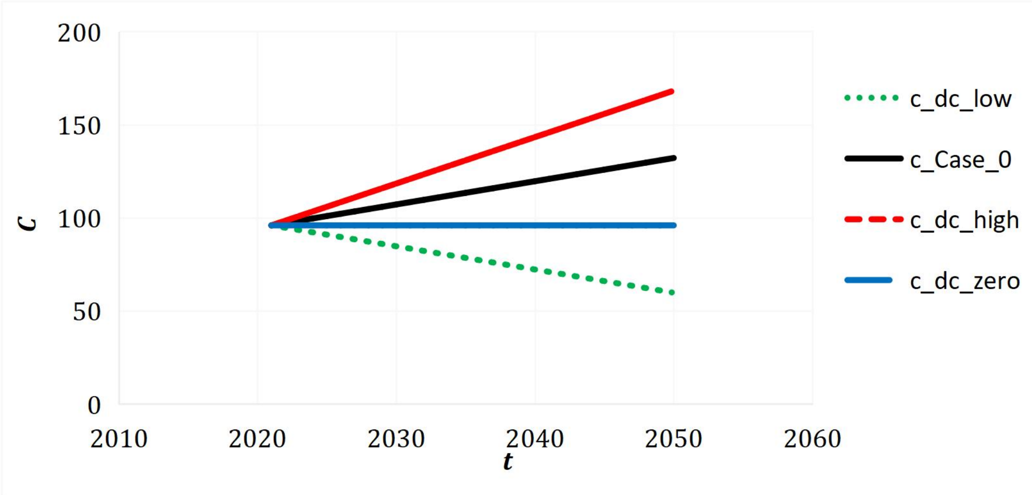

Figure 19. The future time paths of C from the year 2021 to 2050, under four alternative assumptions:

c_dc_low (green): C decreases. The trend is (−1) x the trend during the period 1993 to 2021.
c_Case 0 (black): C increases according to the trend during the period 1993 to 2021.
c_dc_high (red): C increases according to 2 × the trend during the period 1993 to 2021.
c_dc_zero (blue): C stays constant at the level of 2021.

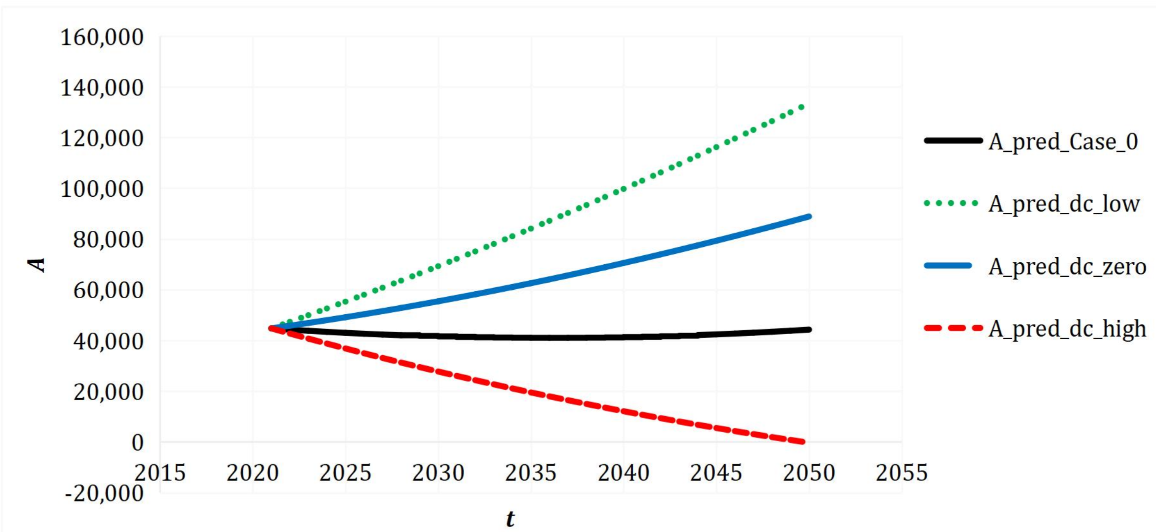

Figure 20. The future time paths of A from the year 2021 to 2050, under four alternative assumptions. Compare Figure 19, where the alternative assumptions are illustrated.

c_dc_low (green): C decreases. The trend is (−1) x the trend during the period 1993 to 2021.
c_Case 0 (black): C increases according to the trend during the period 1993 to 2021.
c_dc_high (red): C increases according to 2 × the trend during the period 1993 to 2021.
c_dc_zero (blue): C stays constant at the level of 2021.

4.2. Summary of the Prediction Results

The graphs in Figures 19 and 20 show that:

- If the real construction cost index grows with the same average time trend as from the year 1993 to 2021, the future apartment construction growth rate will stay almost constant at 40,000 apartments per year until 2050.

- If the future real construction cost index stays constant at the 2022 level, the production of new apartments will grow to almost 90,000 apartments per year in 2050.
- If the real construction cost index can be decreased to the 1993 level, the production of new apartments will grow to almost 130,000 apartments per year in 2050.
- If the future real construction cost index grows two times more rapidly than the average level from 1993 to 2021, the production of apartments will stop completely in 2050.

5. Discussion

The initial mathematical modeling assumptions in this paper motivate the hypotheses concerning the properties of the function that describes how the constructed number of apartments depends on the total budget, represented by the gross national product. Hypothesis 1, used in the regression analysis, states that the number of produced apartments should be a strictly increasing and strictly convex function of GNP. Basic assumptions are that the utility of living space, S, is a strictly concave function of S, with a strictly positive third derivative, and that the utility of other investments is a strictly concave quadratic function of these other investments. The signs of third derivatives of functions sometimes turn out to be of fundamental importance to optimal decision-making. This is particularly relevant and true in multi-period models when stochastic processes and adaptive decisions are optimized, as reported by Lohmander (1988) [13]. The author encourages future research in the construction area to be directed toward multi-period optimization and sequential decision-making under risk. Lohmander (2018) [14] describes alternative adaptive stochastic dynamic optimization methods that may be useful in this context. In this research, it is important to explicitly handle risk in predictions of future demand, prices, GNP, immigration, and other factors of relevance.

Limitations of the Results:

With a limited number of empirical observations, it is not possible to obtain a perfect model. Still, the statistical tests indicate that the developed model can explain the apartment construction dynamics. The present analysis is based on the existing observations in the official statistical data series. Over time, more statistical observations will most likely become available, and further improve the results.

6. Conclusions

The production of new apartments in Sweden has varied strongly during the period 1975 to 2021. A new statistical function, which explains these production changes, has been developed. This function is designed based on a set of hypotheses of how the production level should be affected by different explaining factors: the GNP, size of the population, growth of the population, and the cost of construction. These hypotheses could not be rejected: the apartment production is a strictly increasing and strictly convex function of GNP, and a strictly increasing function of the size of the population, and the growth of the population. The apartment production is a strictly decreasing function of the real cost of construction. The parameters of the statistical function have been estimated with high precision via multiple regression analysis. It was not possible to detect heteroscedasticity via the residual analysis. Furthermore, no indications that nonlinear transformations would improve the selected model were found. The apartment production also contains a strongly significant negative time trend.

Economic Policy Implications:

The estimated function is used to predict the future apartment production until 2050. The predictions are based on assumed growth levels of GNP and the population, and on alternative levels of the cost of construction. If the real construction cost index grows with the same average trend as from 1993 to 2021, the future apartment construction will stay almost constant at 40,000 apartments per year until 2050. If the future real construction cost index stays constant at the 2022 level, the production of new apartments will grow to almost 90,000 apartments per year in 2050. If the real construction cost index can be decreased to the 1993 level, the production of new apartments will grow to almost 130,000 apartments per year in 2050. If the future real construction cost index increases two times more rapidly than the average level from 1993 to 2021, the production of apartments will stop completely in 2050.

Funding

This research obtained no funding.

Institutional Review Board Statement

Not applicable.

Informed Consent Statement

Not applicable.

Data Availability Statement

The empirical data is found in the references.

Conflict of Interests

The author confirm no conflict of interests.

Appendix A

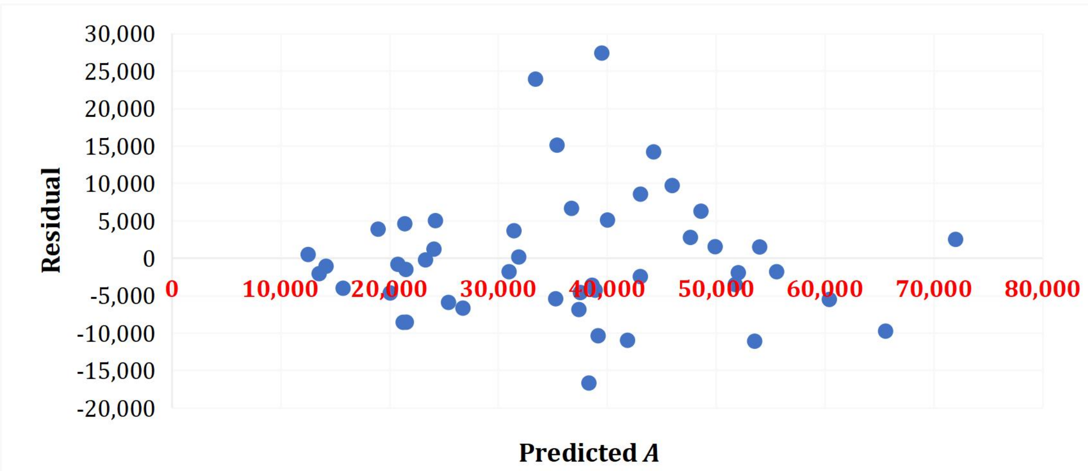

Figure A1. The residuals for different values of A.

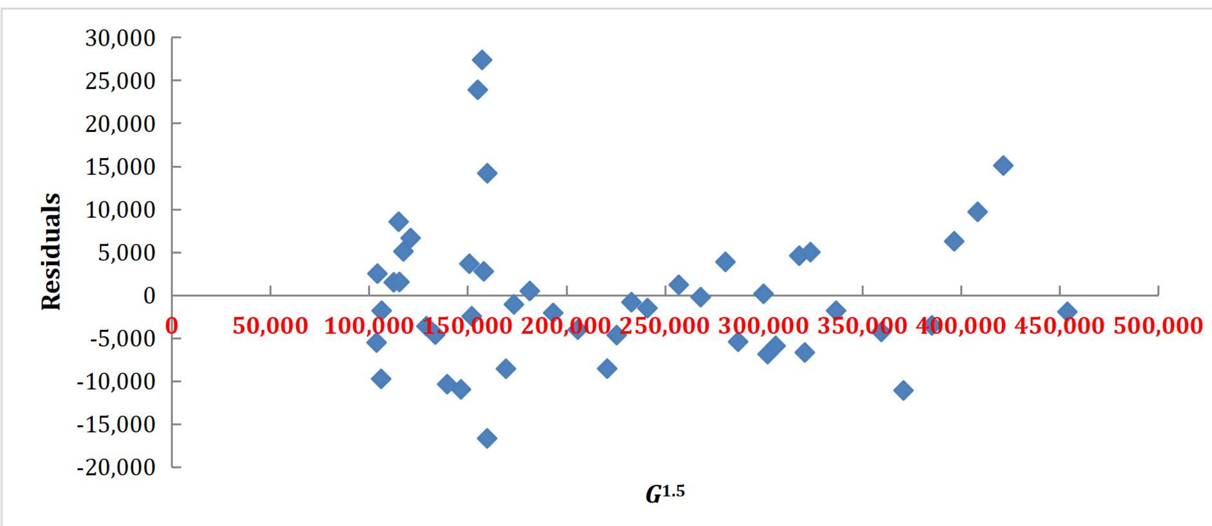

Figure A2. The residuals for different values of $G^{1.5}$.

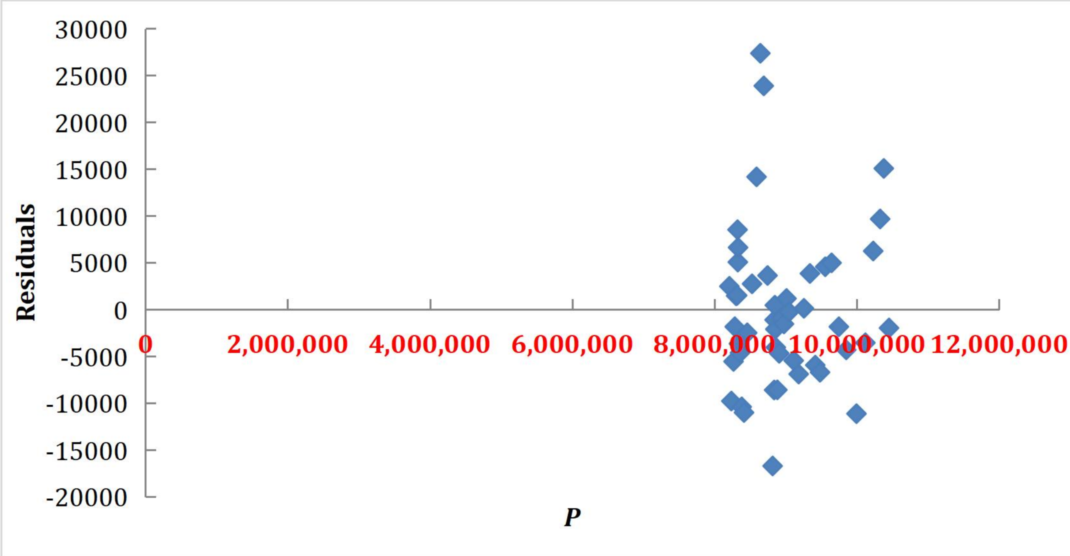

Figure A3. The residuals for different values of *P*.

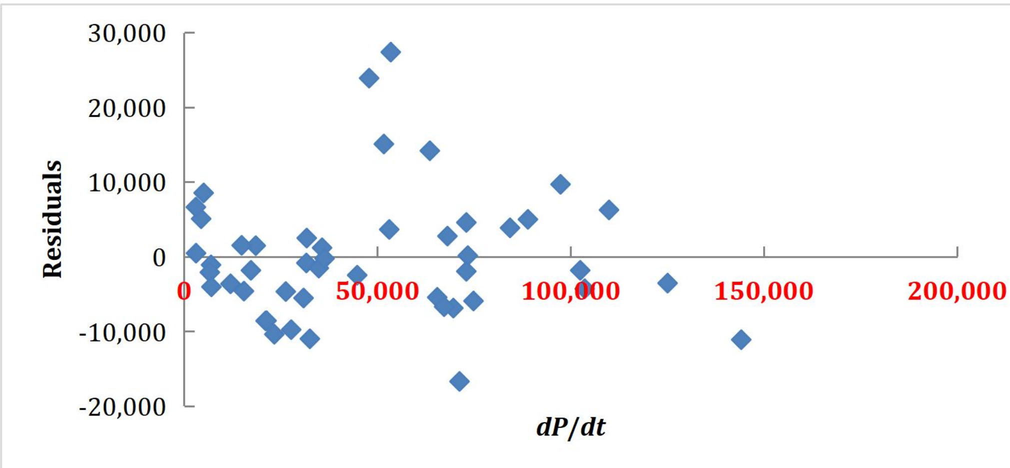

Figure A4. The residuals for different values of dP/dt.

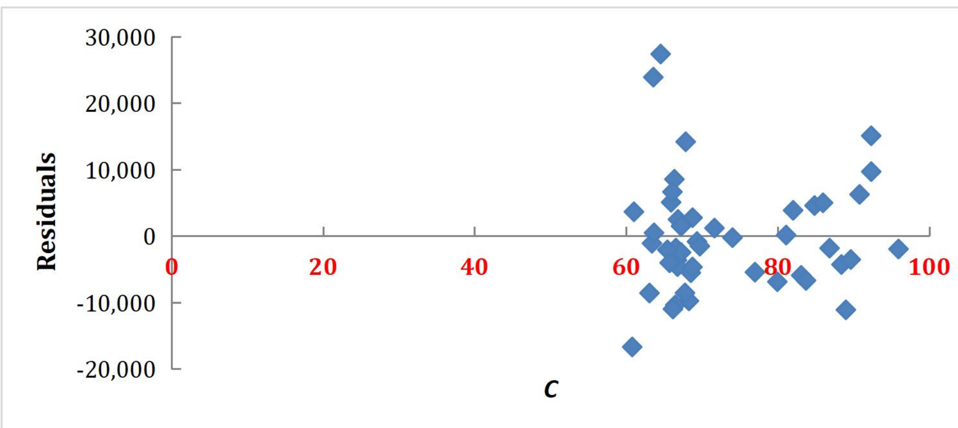

Figure A5. The residuals for different values of *C*.

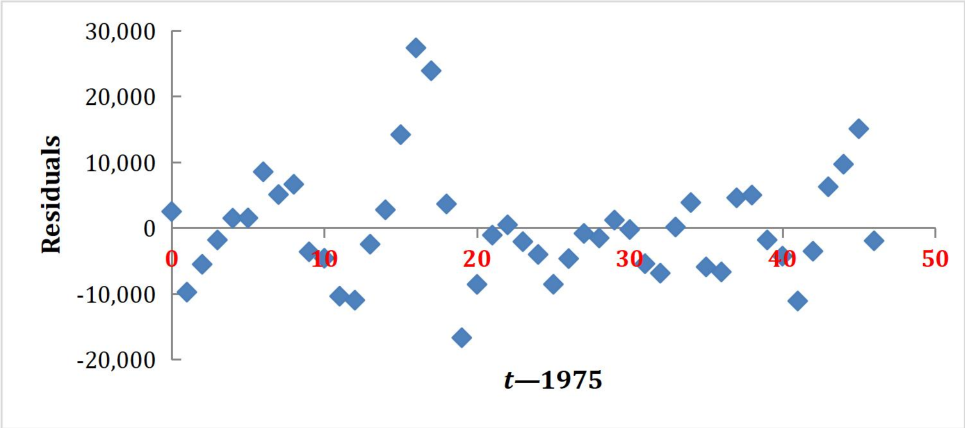

Figure A6. The residuals for different values of *t*—1975.

Appendix B

The definitions of the variables and presentation of the empirical data, are found in the main text. Source: Statistics Sweden (2023).

Table B1. Descriptive statistics.

	Year	*A*	*P*	*dP/dt*	*C*	GNP
Mean	1998	36127.85	8993132	48417.77	73.37629	3621.531
Median	1998	32932	8854322	36360	68.2815	3344.14
Maximum	2021	74499	10452326	144136	95.94208	5906.016
Minimum	1975	11459	8208442	3089	60.796	2210.225
Std. Dev.	13.71131	16707.63	650383.8	35531.61	10.18412	1099.243
Skewness	−2E-17	0.23895	0.800173	0.69933	0.801421	0.426499
Kurtosis	−1.2	−0.96638	−0.34621	−0.15849	−0.83774	−1.09272
Observations	47	47	47	47	47	47

Table B2. Correlation matrix.

	Year	*A*	*P*	*dP/dt*	*C*	GNP
Year	1					
A	−0.30908	1				
P	0.958125	−0.09215	1			
dP/dt	0.724773	0.163363	0.767119	1		
C	0.838958	0.099366	0.911233	0.787388	1	
GNP	0.982348	−0.18076	0.976011	0.753562	0.915475	1

As we see in Table A2, some correlations are rather high. Still, multicollinearity did not pose any problems for the statistical estimations and conclusions derived from these estimations.

Table B3. The empirical data.

t (Year)	*A*	*P*	*dP/dt*	*C*	GNP
1975	74499	8208442	31751	66.83398115	2216.138632
1976	55812	8236179	27737	68.28149691	2242.858762
1977	54878	8267116	30937	68.53278919	2210.225093
1978	53742	8284437	17321	66.56027494	2246.290324
1979	55491	8303010	18573	67.24764567	2331.622388
1980	51438	8317937	14927	67.25544452	2371.593973
1981	51597	8323033	5096	66.3608999	2366.069763

1982	45108	8327484	4451	65.92952758	2399.114127
1983	43364	8330573	3089	66.10466203	2448.49135
1984	34988	8342621	12048	66.70035943	2554.938619
1985	32932	8358139	15518	66.7747558	2614.376693
1986	28791	8381515	23376	66.4786596	2692.052729
1987	30885	8414083	32568	66.17562422	2780.955232
1988	40575	8458888	44805	67.24683808	2849.576978
1989	50404	8527036	68148	68.76075862	2925.313488
1990	58447	8590630	63594	67.8529831	2947.271449
1991	66886	8644119	53489	64.54892803	2914.765974
1992	57319	8692013	47894	63.58350079	2887.499834
1993	35088	8745109	53096	61.02552406	2834.699358
1994	21630	8816381	71272	60.7959968	2946.102273
1995	12678	8837496	21115	63.07506871	3062.037081
1996	13085	8844499	7003	63.40293185	3110.400622
1997	13007	8847625	3126	63.67698123	3205.9063
1998	11459	8854322	6697	65.43700349	3344.139733
1999	11712	8861426	7104	65.7340921	3486.171506
2000	12984	8882792	21366	67.76073365	3652.334602
2001	15411	8909128	26336	68.74737917	3705.275142
2002	19941	8940788	31660	69.34087777	3786.677112
2003	19986	8975670	34882	69.72389657	3874.142181
2004	25283	9011392	35722	71.65150036	4042.158154
2005	23068	9047752	36360	74.0307931	4157.715502
2006	29832	9113257	65505	76.9944268	4351.580361
2007	30527	9182927	69670	79.95140125	4501.240861
2008	32021	9256347	73420	81.11014314	4480.960103
2009	22821	9340682	84335	82.03247468	4286.495819
2010	19500	9415570	74888	83.09153847	4541.632637
2011	20064	9482855	67285	83.73050003	4686.752934
2012	25993	9555893	73038	84.85780416	4659.180553
2013	29225	9644864	88971	85.9802464	4714.521168
2014	29164	9747355	102491	86.8413879	4839.82363
2015	34603	9851017	103662	88.40646127	5057.096941
2016	42441	9995153	144136	88.99603986	5161.808844
2017	48227	10120242	125089	89.64463278	5294.360199
2018	54876	10230185	109943	90.8000308	5397.601433
2019	55659	10327589	97404	92.33861108	5504.80837
2020	50479	10379295	51706	92.33587829	5621.144619
2021	50089	10452326	73031	95.94208476	5906.015621
2022				100	

References

1. Gat, D. Optimal Development of a Building Site. *J. Real Estate Finance Econ.* **1995**, *11*, 77–84. [CrossRef]
2. Dipasquale, D. Why Don't We Know More about Housing Supply? *J. Real Estate Finance Econ.* **1999**, *18*, 9–23. [CrossRef]
3. Pyhrr, S.; Roulac, S.; Born, W. Real Estate Cycles and Their Strategic Implications for Investors and Portfolio Managers in the Global Economy. *J. Real Estate Res.* **1999**, *18*, 7–68. [CrossRef]
4. Witkiewicz, W. The Use of the HP-Filter in Constructing Real Estate Cycle Indicators. *J. Real Estate Res.* **2002**, *23*, 65–88. [CrossRef]
5. Wigren, R.; Wilhelmsson, M. Construction Investments and Economic Growth in Western Europe. *J. Policy Modell.* **2007**, *29*, 439–451. [CrossRef]
6. Batrancea, L.; Rathnaswamy, M.M.; Batrancea, I. A Panel Data Analysis of Economic Growth Determinants in 34 African Countries. *J. Risk Finance Manage.* **2021**, *14*, 260. [CrossRef]
7. Ott, S.H.; Hughen, W.K.; Read, D.C. Optimal Phasing and Inventory Decisions for Large-Scale Residential Development Projects. *J. Real Estate Finance Econ.* **2012**, *45*, 888–918. [CrossRef]

8. Guthrie, G. Land Hoarding and Urban Development. *J. Real Estate Finance Econ.* **2023**, *67*, 753–793. [CrossRef]
9. Barr, J. Skyscraper Height. *J. R. Estate Finance Econ.* **2012**, *45*, 723–753. [CrossRef]
10. Barr, J.; Luo, J. Growing Skylines: The Economic Determinants of Skyscrapers in China. *J. R. Estate Finance Econ.* **2021**, *63*, 210–248. [CrossRef]
11. Koblyakova, A.; Fleishman, L.; Furman, O. Accuracy of Households' Dwelling Valuations, Housing Demand and Mortgage Decisions: Israeli Case. *J. Real Estate Finance Econ.* **2022**, *65*, 48–74. [CrossRef]
12. Statistics Sweden: Available online at: https://www.scb.se/en/ (accessed on 14 June 2024).
13. Lohmander, P. Continuous Extraction Under Risk. *Syst. Anal.-Modell.-Semin.* **1988**, *5*, 131–151.
14. Lohmander, P. Applications and Mathematical Modeling in Operations Research. In Proceedings of the International Workshop on Mathematics and Decision Science, Guangzhou, China, 12–15 September, 2016. [CrossRef]

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