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Article

Empirical Study on the Influence of Different Mathematical Methods on Chat GPT (AI) Competence in Solving Quadratic Root Functions

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Abstract: This empirical study investigates the impact of two distinct mathematical problem-solving methods – the Algebraic Formula Method and the Newton Sum Method – on enhancing Chat GPT's competence in effectively solving quadratic root functions. The integration of Artificial Intelligence (AI) into mathematical problem-solving has paved the way for innovative approaches. In this study, we delve into the Algebraic Formula Method and the Newton Sum Method, essential techniques for solving quadratic root functions. We aim to showcase the profound influence of these methods on Chat GPT's capacity to excel in solving quadratic equations. Through concrete evidence, we demonstrate Chat GPT's adept utilization of the Newton Sum Method for quadratic root function calculations. While Chat GPT can compute quadratic root functions of the form addition $\alpha \wedge 15 + \beta \wedge 15$ using this method, its proficiency in using algebraic formula methods typically extends only up to addition $\alpha \wedge 4 + \beta \wedge 4$. This marked discrepancy underscores the pivotal role that different methods play in amplifying the AI system's mathematical capabilities. Chat GPT excels in solving quadratic equations using the Newton Sum Method compared to the Algebraic Formula Method. The model adeptly computes expressions of the form addition $\alpha \wedge 15 + \beta \wedge 15$ using this method, while its proficiency using algebraic formula methods is generally limited to addition $\alpha \wedge 4 + \beta \wedge 4$. This striking discrepancy underscores the transformative impact that different methods can have on elevating the AI system's mathematical prowess. The research value is Pioneering Novel Maths Approaches for Overcoming Limitations in AI. This study serves as an illuminating testament to the significance of pioneering innovative methodologies, rules, theorems, or formulas to surmount the current limitations in AI systems like Chat GPT. These innovative pursuits hold the key to unlocking the untapped potential that lies within, propelling AI systems to greater heights of proficiency. In essence, they offer a strategic pathway towards expanding the capabilities of AI and pushing the boundaries of what can be achieved.

Keywords: Algebraic Formula Method; Newton Sum; AI System; Chat GPT; Vieta Theorem; Quadratic Root Functions

1. Introduction

This empirical study investigates the impact of two distinct mathematical problem-solving methods – the Algebraic Formula Method and the Newton Sum Method – on enhancing Chat GPT's competence in effectively solving quadratic root functions.

1.1ChatGPT[1]:

Meet Chat GPT, the latest conversational AI model designed for interactive dialogue. Unlike traditional interfaces, Chat GPT engages users with the ability to answer follow-up questions, acknowledge errors, challenge flawed assumptions, and decline inappropriate requests.

Chat GPT[2] Plus: , an advanced conversational AI that can engage in meaningful dialogue, provide follow-up answers, and challenge misconceptions. While its knowledge is currently based on data up to 2021, it excels at handling interactive conversations.

1.2 Knowledge is Power

1.2.1 Why the future is not just about the tech[3]

While technology plays a crucial role, it's only one aspect of the equation. Human intelligence is poised to be one of the most significant assets in the current Fourth Industrial Revolution (FIR). Companies risk struggling if they do not achieve the right balance between automated technology and human insights.

1.2.2 Data Is the Backbone [4]

Designed with simplicity in mind, Ricoh Supervisor is user-friendly for everyday individuals, including operation managers and other decision-makers. We understand that complex data, charts, and graphs can be intimidating. With Ricoh Supervisor, you don't need to be a data analyst to navigate and utilize the software effectively.

1.3 Algebra Formula

1.3.1 The following functions are some symmetric function of a quadratic's roots[5],

i) $\alpha ^{2} + \beta ^{2} = (\alpha + \beta) ^{2} - 2\alpha\beta$ ii) $(\alpha - \beta) ^{2} = (\alpha + \beta) ^{2} - 4\alpha\beta$, iii) $\alpha ^{3} + \beta ^{3} = (\alpha + \beta) [(\alpha + \beta) ^{2} - 3\alpha\beta]$ iv) $\alpha ^{4} + \beta ^{4} = (\alpha ^{2} + \beta ^{2})^{2} - 2(\alpha ^{2}) (\beta ^{2})$

1.3.2 The following functions are asymmetric function of a quadratic's roots[5],

i) $\alpha \wedge 2 - \beta \wedge 2 = \pm (\alpha + \beta)[(\alpha + \beta) \wedge 2 - 4\alpha\beta]$ ii) $\alpha \wedge 3 - \beta \wedge 3 = (\alpha - \beta)[\alpha \wedge 2 + \beta \wedge 2 + \alpha\beta]$

Note: Student need to remember above Algebra Formula if they use Veda's theorem method for Construct New Quadratic Equation.

1.4 Newton Sum (Newton's Identities)[6]

Newton's identities or newton- Girard formulae, provide a method to compute the power sums of the roots of a polynomial without explicitly determining the roots themselves. If x_1, x_2, \dots, x_n are the roots of a polynomial equa-

tion,, these identities allow us to find the sums

$$\sum_{i=1}^{n} x_{i^{k}} = x_{1^{k}} = x_{1$$

1.5 The French Mathematician, Vieta [7].



Figure 1 : François Vièta (Latin: Franciscus Vieta; 1540 - 23 February 1603) was a notable French mathematician who made significant contributions to algebra. In 1615, Vièta established a fundamental relationship between the roots and coefficients of equations.

Vièta's theorem, specifically for quadratic equations $a \ge x^2 + bz + c = 0$ where $a \ne 0$, states the following:

If α and β a are the roots of the quadratic equation $a \ge a + b = -b/a$, $\alpha + \beta = -b/a$, $\alpha \beta = c/a$

These relationships between the roots f α and β are and the coefficients ,a,b, and c are foundational in algebraic theory. Vièta's work laid the groundwork for further developments in algebra and polynomial equations, influencing mathematicians for centuries.

Prove[8,9] $\alpha + \beta = -\mathbf{b}/\mathbf{a}$ and $\alpha\beta = \mathbf{c}/\mathbf{a}$. If α and β are two roots of the quadratic equation a $x^2 + \mathbf{b}x + \mathbf{c} = \mathbf{0}$ and $a \neq 0$. Then $\alpha = [-\mathbf{b} + \sqrt{\mathbf{b}^2 - 4ac}]/(2a)$, $\beta \equiv [-\mathbf{b} - \sqrt{\mathbf{b}^2 - 4ac}]/(2a)$ So, $\alpha + \beta = [-\mathbf{b} + \sqrt{\mathbf{b}^2 - 4ac}]/(2a) + [-\mathbf{b} - \sqrt{\mathbf{b}^2 - 4ac}]/(2a)$ = (-2b)/(2a) = -b/a $\alpha \beta = [-\mathbf{b} + \sqrt{\mathbf{b}^2 - 4ac}]/(2a) \times [-\mathbf{b} - \sqrt{\mathbf{b}^2 - 4ac}]/(2a)$ $= [(-b)^2 - (\sqrt{\mathbf{b}^2 - 4ac})^2]/(2a)^2$ $= [(-b)^2 - (\sqrt{\mathbf{b}^2 - 4ac})^2]/(2a)^2$ $= [(-b)^2 - ((\sqrt{\mathbf{b}^2 - 4ac})^2)]/(4a^2)]$ $= \frac{(4ac)}{(4a^2)}$ = c/a

1.6 Advanced mathematical techniques such as Peter Chew Rule to AI systems like ChatGPT to overcome limitation[10] and error **[11]** Problem

In addition to applying Peter Chew Rule in the Education 4.0 [12] calculator, Peter Chew Rule are also applicable to many different Engineering fields, such as Civil Engineering, please refer to article Peter Chew Rule for solution of Triangle**[13]**, Mechanical **Engineering[14]**, Electrical **Engineering[15]**, Aerospace Engineering[16], Marine Engineering [17] and Astronomical Engineering[18]. Furthermore, Peter Chew Rule are also applicable to Pool Game[19] and Criminology[20]

1.7 Advanced mathematical techniques such as Peter Chew theorem to AI systems like ChatGPT to overcome limitation GPT[21] Problem

In addition, Peter Chew Theorem also applicable to many different Engineering fields, such as Civil Engineering [22], Mechanical Engineering [23, 24], Electrical Engineering [25], Aerospace Engineering [26], Marine Engineering [27] and Astronomical Engineering [28]. Furthermore, Peter Chew Theorem are also applicable to Pool Game [29] and Criminology [30].

2. Evidence

Through concrete evidence, we demonstrate Chat GPT's adept utilization of the Newton Sum Method for quadratic root function calculations. While Chat GPT can compute quadratic root functions of the form $\alpha ^{15+\beta} ^{15}$ using this method, its proficiency in using algebraic formula methods typically extends only up to $\alpha ^{14+\beta} ^{4}$. This marked discrepancy underscores the pivotal role that different methods play in amplifying the AI system's mathematical capabilities

2.1 Algebraic Formula Methods.

2.1.1 Using Chat *GPT* for Calculate If the roots of the equation $x \wedge 2 - 3x + 2 \equiv 0$ are α and β , find $\alpha \wedge 2 + \beta \wedge 2$ is shown in figure 2.

It the roots of the equation x^{*2} -3x $^{+2}$ = 0 are α and β , find the value of α *2 + β^{*2}
The sum of the roots $\alpha+\beta$ is equal to the negation of the coefficient of x term divided by the coefficient of the x^2 term:

Figure 2: Chat GPT solution on if the roots of the equation $x \wedge 2 - 3x + 2 \equiv 0$ are α and β , find $\alpha \wedge 2 + \beta \wedge 2$ 2.1.2 Algebra Formula, Using Chat *GPT* for Calculate If the roots of the equation $x \wedge 2 - 3x + 2 \equiv 0$ are α and β , find $\alpha \wedge 3 + \beta \wedge 3$ is shown in figure 3,



Figure 3: Chat GPT solution on If the roots of the equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find $\alpha \wedge 3 + \beta \wedge 3$ 2.1.3 Algebra Formula, Using Chat *GPT* for Calculate If the roots of the equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find $\alpha \wedge 4 + \beta \wedge 4$ is shown in figure 4,

If the roots of the equation x^2 -3x +2 = 0 are α and β , find the value of α ^4+ β ^4
The value of $\alpha^4 + \beta^4$ can be found using the following reation: $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$. In your equation $x^2 - 3x + 2 = 0$, we've already calculated $\alpha^2 + \beta^2$ as 5. We can also calculate the product of the squares of the roots $\alpha^2\beta^2$, which is equal to $(\alpha\beta)^2$ (product of the roots) squared: $\alpha^2\beta^2 = (\alpha\beta)^2 = (2)^2 = 4$. Now, we can plug these values into the formula: $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = 5^2 - 2 \cdot 4 = 25 - 8 = 17$. So, the value of $\alpha^4 + \beta^4$ is 17.

Figure 4: Algebra Formula, Using Chat *GPT* for Calculate If the roots of the equation $x \wedge 2 - 3x + 2 \equiv 0$ are α and β , find $\alpha \wedge 4 + \beta \wedge 4$ is shown in figure 45,

2.1.4 Algebra Formula, Using Chat *GPT* for Calculate If the roots of the equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find $\alpha \wedge 5 + \beta \wedge 5$ is shown in figure 5,



Figure 5: Chat GPT solution on If the roots of the equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find $\alpha \wedge 5 + \beta \wedge 5$

When employing the Algebraic Formula, Chat GPT faces limitations in delivering conclusive solutions for problems like $\alpha \wedge 5 + \beta \wedge 55$, owing to the intricate nature of the solutions. As we see Chat GPT Reply : At this point, finding the exact value of $\beta \wedge 2$ and subsequently $\alpha \wedge 5 + \beta \wedge 5$ becomes more complex and likely involves solving a higher-degree equation for β . The process becomes intricate and might not lead to a simple numerical result. In general, the expressions involving higher powers of the roots can become quite involved, and finding specific numerical values can be complex.

2.2 Newton sum Methods.

2.2.1 If the roots of the equation $\mathbf{x} \wedge \mathbf{2} - 3\mathbf{x} + 2 = 0$ are α and β , find the value of $\alpha \wedge \mathbf{2} + \beta \wedge \mathbf{2}$ using Newton sum . Sk = $\alpha \wedge \mathbf{k} + \beta \wedge \mathbf{k} = -\mathbf{a}_{(k-1)} * \mathbf{S}_{(k-1)} - \mathbf{a}_{(k-2)} * \mathbf{S}_{(k-2)}$ is shown in figure 6.

If the roots of the equation x^2 - 3x + 2 = 0 are α and β , find the value of $\alpha^2 + \beta^2$ using Newton sum. Sk = $\alpha^k + \beta^k = -a_0(c-1) + S_0(c-1) - a_0(c-2) + S_0(c-2)$
\$ Given the quadratic equation $x^2 - 3x + 2 = 0$, we have α and β as its roots. The sum of the roots is $\alpha + \beta = 3$ and the product of the roots is $\alpha\beta = 2$.
We want to find the value of a^+ + β^- , which can be expressed using Newton's sums as: $S_2=a^2+\beta^2=-a_1\cdot S_1-a_0\cdot S_{0,}$
where a_1 is the coefficient of the linear term (which is -3 in this case), a_0 is the constant term (which is 2), S_1 is the sum of the roots (which is 3), and S_0 is the number of roots (which is 2).
Plugging in the values: $S_2=-(-3)\cdot 3-2\cdot 2=9-4=5.$
So, the value of $lpha^2+eta^2$ is 5.

Figure 6: Chat GPT solution on If the roots of the equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find the value of $\alpha \wedge 2 + \beta \wedge 2$ using Newton sum . Sk = $\alpha \wedge k + \beta \wedge k = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$.

2.2.2 If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^3 + \beta^3$ using Newton sum . Sk = $\alpha^k + \beta^k = -a(k-1) * S(k-1) - a(k-2) * S(k-2)$ is shown in **figure 7**.

R	If the roots of the equation $x'2^{-3}x+2=0$ are α and β , find the value of $\alpha''3+\beta''3$ using Newton sum. Sk-ak+βk=-al(DSk-1+cSk-2)
Ø	Certainly, to find the value of $\alpha^*3+\beta^*3$ using Newton's sum, we can use the formula you've $\hfill G$ provided:
	$Sk = \alpha^{*}k + \beta^{*}k = -a_{-}(1) * S_{-}(k-1) - a_{-}(2) * S_{-}(k-2)$
	Given the equation $x^2 - 3x + 2 = 0$, the coefficients are: a. $2 = 1$
	a_1=-3 a_0=2
	Let's use Newton's sum to find $\alpha'3$ + $\beta'3:$
	1. S ₂ 0=α [*] 0+β [*] 0=2
	2. S_1=α*1+β*1=-(-3)=3
	Now we can use the formula to find S_2:
	$S_2 = \alpha^2 + \beta^2 = -a_1 + S_1 - a_0 + S_0$
	=-{-3}*3-2*2 =9-4
	-5
	Now let's use the formula again to find S_3:
	S_3=α*3+β*3=-a_1*S_2-a_0*S_1
	=-(-3)*5-2*3
	= 3 There was an error generating a response

Figure 7: Chat GPT solution on If the roots of the equation $\mathbf{x} \cdot \mathbf{2} - 3\mathbf{x} + 2 = 0$ are α and β , find the value of $\alpha \cdot \mathbf{3} + \beta \cdot \mathbf{3}$ using Newton sum . Sk = $\alpha^{k} + \beta^{k} = -\mathbf{a}(k-1) \cdot \mathbf{s}(k-1) - \mathbf{a}(k-2) \cdot \mathbf{s}(k-2)$.

2.2.3 If the roots of the equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find the value of $\alpha \wedge 4 + \beta \wedge 4$ using Newton sum . Sk = $\alpha \wedge k + \beta \wedge k = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$ is shown in **figure 8**.



Figure 8: Chat GPT solution on If the roots of the equation $x \wedge 2 - 3x + 2 =$ equation $x \wedge 2 - 3x + 2 = 0$ equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find the value of $\alpha \wedge 5 + \beta \wedge 5$ using Newton sum. Sk = $\alpha \wedge k + \beta \wedge k = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$.

2.2.5 If the roots of the equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find the value of $\alpha \wedge 6 + \beta \wedge 6$ using Newton sum . Sk = $\alpha \wedge k + \beta \wedge k = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$ is shown in **figure 10**.



Figure 10: Chat GPT solution on If the roots of the equation $\mathbf{x}^2 - 3\mathbf{x} + 2 = 0$ are α and β , find the value of $\alpha^{6} + \beta^{6}$ using Newton sum . Sk = $\alpha^{k} + \beta^{k} = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$.

2.2.6 If the roots of the equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find the value of $\alpha \wedge 7 + \beta \wedge 7$ using Newton sum . Sk = $\alpha \wedge k + \beta \wedge k = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$ is shown in figure 11.

If the roots of the equation $x^{*}2^{-3}x + 2 = 0$ are α and β , find the value of $\alpha^{*}7 + \beta^{*}7$ using	Now we use the formula again to find S_4:
Newton sum. SK=ak+pk=-aluSk=HCSk=z)	$S_4 = \alpha^4 + \beta^4 = -a_1 + S_3 - a_0 + S_2$
	= -{-3} *9 - 2 *5
To find the value of a"7 + B"7 using Newton's sums, we'll use the following formula:	= 27 - 10
	= 17
$SK = \alpha^{n}K + \beta^{n}K = -a_{(1)} * S_{(K-1)} - a_{(2)} * S_{(K-2)}$	
	Now we use the formula again to find S_5:
Given the equation x 2 - 3x + 2 = 0, the coefficients are:	
a.2=1	S_5=α^5+β^5=-a_1*S_4-a_0*S_3
8_1=-0	= -(-3)*17 - 2*9
a_0=2	= 51 - 18
Let's use Newton's sum to find $\alpha''7 + \beta''7$:	= 33
1. $S_0 = \alpha^0 + \beta^0 = 2$	Now we use the formula again to find S_6:
2. $S_1 = \alpha^{n} + \beta^{n} = -(-3) = 3$	S 6 = 0°6 + 8°6 = -a 1*S 5 - a 0*S 4
Now we can use the formula to find 9, 9-	(3)*33.0*17
	- 00 - 24
$S_2 = \alpha^2 + \beta^2 = -a_1 + S_1 - a_0 + S_0$	-33-04
= -(-3)*3-2*2	- co
= 9 - 4	Now we use the formula again to find \$ 7:
= 5	· · · · · · · · · · · · · · · · · · ·
	$S_7 = \alpha^7 + \beta^7 = -a_1 S_6 - a_0 S_5$
Now we use the formula again to find 5_3.	= -(-3) * 65 - 2 * 33
S_3=α^3+β^3=-a_1*S_2-a_0*S_1	= 195 - 66
=-(-3)*5-2*3	= 129
= 15 - 6	
= 9 There was an error generating a response	So, the value of α 7 + β 7 is 129.

Figure 11: Chat GPT solution on If the roots of the equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find the value of $\alpha \wedge 7 + \beta \wedge 7$ using Newton sum . Sk = $\alpha \wedge k + \beta \wedge k = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$.

2.2.7 If the roots of the equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find the value of $\alpha \wedge 8 + \beta \wedge 8$ using Newton sum . Sk = $\alpha \wedge k + \beta \wedge k = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$ is shown in figure 12.





Figure 12: Chat GPT solution on If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^8 + \beta^8$ using Newton sum . Sk = $\alpha^k + \beta^k = -a(k-1) + S(k-1) - a(k-2) + S(k-2)$.

2.2.8 If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^2 + \beta^2 + \beta^2 - 3x + 2 = 0$ using Newton sum . Sk = $\alpha^k + \beta^k = -a_{(k-1)} + S_{(k-1)} - a_{(k-2)} + S_{(k-2)}$ is shown in **figure 13**.



Figure 13: Chat GPT solution on If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^9 + \beta^9$ using Newton sum . Sk = $\alpha^k + \beta^k = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$.

2.2.9 If the roots of the equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find the value of $\alpha \wedge 10 + \beta \wedge 10$ using Newton sum . Sk = $\alpha \wedge k + \beta \wedge k = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$ is shown in **figure 14**.





Figure 14: Chat GPT solution on If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{10} + \beta^{10}$ using Newton sum . Sk = $\alpha^{k} + \beta^{k} = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$.

2.2.10 If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{11} + \beta^{11}$ using Newton sum . Sk = $\alpha^{k} + \beta^{k} = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$ is shown in figure 15



Figure 15: Chat GPT solution on If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{11} + \beta^{11}$ using Newton sum . Sk = $\alpha^{k} + \beta^{k} = -a(k-1) * S(k-1) - a(k-2) * S(k-2)$.

2.2.11 If the roots of the equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find the value of $\alpha \wedge 12 + \beta \wedge 12$ using Newton sum . Sk = $\alpha \wedge k + \beta \wedge k = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$ is shown in figure 16

$ \begin{array}{ $		in the roots of the equation × 2+ αx+2+ σ are u and p, ind the value of u 12+p 12 usin Newton sum. Sk=ck+βk=-a1(bSk-1+cSk-2)	
To increase use of $r^{12} + q^{12} users (weetons aure, will use to indexing termule: (a) q^{12} + q^{12} + q^{12} + q^{12} (q^{12} + q^{12} + $			$S_4 = \alpha^4 + \beta^4 = -a_1 + S_3 - a_0 + S_2$
			=-(-3)*9-2*5
$ \begin{array}{ c c } & = -77 \\ & = $	\$	To find the value of α^{12} + β^{12} using Newton's sums, we'll use the following formula:	[□] (= 27-10
$ \frac{1}{2} = \frac{1}{2} + 1$		CIU- ポイレ→ (パレー・a) (パレ・1) - a (ワ) 本○ (ルレワ)	= 17
			Now we use the formula again to find S_5:
$ \begin{array}{c} 32^{-1} \\ 41^{-3} \\ 40^{-2} \\ 40^{-2} \\ 40^{-2} \\ 10^{-1} + 10^{-1} + 6^{-1} + 6^{-1} + 15^{-1} + $		Given the equation x ² - 3x + 2 = 0, the coefficients are:	
$ \frac{1}{2} + 1$		a_2=1	$S_5 = \alpha^5 + \beta^5 = -a_1 + S_4 - a_0 + S_3$
= 51 - 18 $= 51 - 18$ $= 51 - 18$ $= 51 - 18$ $= 53$ Now we use the formula sign to find \$2,6 $= 5 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4$ $= 5 - 4 - 4 - 4$ $= 5 - 4 - 4 - 4$ $= 5 - 4 - 4 - 4 - 4$ $= 5 - 4 - 4 - 4 - 4$ $= 5 - 4 - 4 - 4 - 4 - 4 - 4 - 4 - 4 - 4 -$		a_1=-3	= -(-3) * 17 - 2 * 9
uets use the formula again to find §.9:		a_0=2	= 51 - 18
$1 \cdot 3_{0} \circ c^{0} + p^{0} - \frac{1}{3}$ $2 \cdot 3_{1} + c^{1} + \beta^{1} + -(3) + 3$ Now we can use the formula to find $\frac{5}{5}$. $3 \cdot 1 - c^{0} + \beta^{1} + 2 - a_{1}^{+} + 3_{0} + a_{0}^{-} + 3_{0}^{-}$ $3 \cdot 1 - c^{0} + \beta^{1} + 2 - a_{1}^{+} + 3_{0}^{-$		Let's use Newton's sum to find α 12 + β 12:	= 33
$ \begin{array}{c} 2.91 + \alpha^{-1} + \beta^{-1} + -(2) + 3 \\ \\ Now we can use the formula to find $2,2 \\ 3.9 + \alpha^{-1} + \beta^{+1} + -(2) + 3,0 + 0$		1. S_0=α*0+β*0=2	Now we use the formula again to find S_6:
Now we can use the formula again to find \$.2 $\frac{5}{2} = x^{n} + 2^{n} = -\frac{1}{2} = -\frac{1}{2} = -\frac{1}{2} = \frac{1}{2} =$		2. S_1 = α ^r 1 + β ^r 1 = -(-3) = 3	
Now we use the formula again to find \$_2. $s_2 = a^2 + b^2 = a_1^{+1} s_2^{-1} = a_0^{+1} s_3^{-1} = a_0^{-1} s_3^{-1} = a_0$			S_6=α^6+β^6=-a_1*S_5-a_0*S_4
$ s_{2} = a^{-1} + \frac{1}{2} + 2 = s_{1}^{-1} + \frac{1}{2} +$		Now we can use the formula to find S_2:	=-(-3)*33-2*17
$= \frac{-(3^{\circ} 5^{\circ} 5^{\circ} 2^{\circ} 2}{-9^{\circ} 4}$ $= \frac{-65}{-9^{\circ} 4}$ $= \frac{-65}{-9^{\circ} 4}$ $= \frac{-65}{-9^{\circ} 4}$ Now we use the formula again to find § 3. $= \frac{-(3^{\circ} 5^{\circ} 5^{\circ} 2^{\circ} 3)}{-(3^{\circ} 5^{\circ} 5^{\circ} 2^{\circ} 3)}$ $= \frac{-(3^{\circ} 5^{\circ} 6^{\circ} 2^{\circ} 3)}{-(3^{\circ} 5^{\circ} 5^{\circ} 2^{\circ} 3)}$ $= \frac{-(3^{\circ} 5^{\circ} 6^{\circ} 2^{\circ} 3)}{-(3^{\circ} 5^{\circ} 6^{\circ} 2^{\circ} 3)}$ $= \frac{-(3^{\circ} 5^{\circ} 6^{\circ} 2^{\circ} 3)}{-(3^{\circ} 5^{\circ} 6^{\circ} 2^{\circ} 3)}$ Now we use the formula again to find § 8. $= \frac{-(3^{\circ} 5^{\circ} 7^{\circ} 3)}{-(3^{\circ} 5^{\circ} 7^{\circ} 3)}$ We use the formula again to find § 9. $= \frac{-(3^{\circ} 5^{\circ} 9^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}{-(3^{\circ} 7^{\circ} 2^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}$ $= \frac{-(3^{\circ} 9^{\circ} 9^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}{-(3^{\circ} 7^{\circ} 2^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}$ $= \frac{-(3^{\circ} 9^{\circ} 9^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}{-(3^{\circ} 7^{\circ} 7^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}$ $= \frac{-(3^{\circ} 9^{\circ} 9^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}{-(3^{\circ} 7^{\circ} 7^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}$ $= \frac{-(3^{\circ} 9^{\circ} 9^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}{-(3^{\circ} 7^{\circ} 7^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}$ $= \frac{-(3^{\circ} 9^{\circ} 9^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}{-(3^{\circ} 7^{\circ} 7^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}$ $= \frac{-(3^{\circ} 9^{\circ} 9^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}{-(3^{\circ} 7^{\circ} 7^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}$ $= \frac{-(3^{\circ} 9^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}{-(3^{\circ} 7^{\circ} 7^{\circ} 9^{\circ} - 2^{\circ} 3, 7)}$ $= \frac{-(3^{\circ} 9^{\circ} 9^{\circ} - 2^{\circ} 3, 6)}{-(3^{\circ} 7^{\circ} 7^{\circ} - 2^{\circ} 3, 7)}$ $= \frac{-(3^{\circ} 9^{\circ} 9^{\circ} - 2^{\circ} 3, 7)}{-(3^{\circ} 9^{\circ} - 2^{\circ} 3, 7)}$ $= \frac{-(3^{\circ} 9^{\circ} 9^{\circ} - 2^{\circ} 3, 7)}{-(3^{\circ} 9^{\circ} - 2^{\circ} 3, 7)}$ $= \frac{-(3^{\circ} 9^{\circ} - 2^{\circ} 3, 7)}{-(3^{\circ} - 2^{\circ} - 2^{\circ} 3, 7)}$ $= \frac{-(3^{\circ} 9^{\circ} - 2^{\circ} - 2^{\circ}$		S 2= 0°2+6°2=-a 1*S 1-a 0*S 0	= 99 - 34
$= \frac{9 \cdot 4}{5}$ Now we use the formula again to find §.2. $= \frac{1}{5} \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 2^{-3} = \frac{1}{5} \cdot 5 \cdot 6 = \frac{1}{5} \cdot 5 \cdot 5 = \frac{1}{5} \cdot 5 - 5 = \frac{1}{5} \cdot 5 - 5 = \frac{1}{5} \cdot 5 - 5 = \frac{1}{5} \cdot 5 = \frac{1}{5} \cdot 5 - 5 = \frac{1}{5} \cdot 5 = \frac{1}{5} \cdot 5 - 5 = \frac{1}{5} \cdot 5 = \frac{1}{5} \cdot 5 - 5 = \frac{1}{5} \cdot 5 = \frac{1}{5} \cdot$		=(-3)*3-2*2	= 65
-5 Now we use the formula again to find 8.3 $33 = r33 + p^{2} - a_{1}^{+} 8 \cdot 2 \cdot a_{0} + 8 \cdot 1$ $8 \cdot 7 - r3 + 6 \cdot 2^{+} \cdot 3 = -4 \cdot 3 \cdot 5 \cdot 4 \cdot 3 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = -4 \cdot 3 \cdot 5 \cdot 5 = -4 \cdot 5 \cdot $			
Now we use the formula again to find \$,3 $3.3 = \alpha^{-1} + \beta^{-1} = \alpha_{1} + 3.2 = \alpha_{1} + 3.2 = \alpha_{1} + 3.3 = \alpha_{1$		=5	Now we use the formula again to find S_7:
Now we use the formula again to find \$.3 $3.3 = a^{-3} + b^{-3} = -a_1 + 3.2 + a_0 + 3.1$ $+ (-3) + 5 + 2^{+3} = -a_1 + 3.2 + a_0 + 3.1$ $+ (-3) + 5 + 2^{+3} = -a_1 + 3.2 + a_0 + 3.1$ $+ (-3) + 5 + 2^{+3} = -a_1 + 3.2 + a_0 + 3.1$ $+ (-3) + 5 + 2^{+3} = -a_1 + 3.2 + a_0 + 3.1$ $+ (-3) + 5 + 2^{-3} = -a_1 + 3.2 + a_0 + 3.1$ $+ (-3) + 5 + 2^{-3} = -a_1 + 3.2 + a_0 + 3.1$ $+ (-3) + 5 + 2^{-3} = -a_1 + 3.2 + a_0 + 3.1$ $+ (-3) + 5 + 2^{-3} = -a_1 + 3.2 + a_0 + 3.1$ $+ (-3) + 5 + 2^{-3} = -a_1 + 3.2 + a_0 + 3.1$ $+ (-3) + 5 + 2^{-3} = -a_1 + 3.2 + a_0 + 3.1$ $+ (-3) + 5 + 2^{-3} = -a_1 + 3.2 + a_0 + 3.1$ $+ (-3) + 2^{-3} + 2^{-3} + 3.0 + 3.1$ $+ (-3) + 2^{-3} + 2^{-3} + 3.0 + 3.1$ $+ (-3) + 2^{-3} + 3.1 + 3.0 + 3.1$ $+ (-3) + 2^{-3} + 3.1 + 3.0 + 3.1 + 3.0 + 3.1$ $+ (-3) + 2^{-3} + 3.1 + 3.0 + 3.1 + 3.0 + 3.1 + 3.0 + 3.1 + 3.0 + 3.1 + 3.0 + 3.1 + 3.0 + 3.1 + 3.0 + 3.1 + 3.0 + 3.1 + 3.0 + 3.1 + 3.0 + 3.1 + 3.0 + 3.1 + 3.0 + 3.1 + 3.0 + 3.1 + 3.0 + 3.1 + 3.0 + 3.1 + 3.0 + 3.1 + 3.0 + 3$			9.7 - α ⁴⁷ + P ⁴⁷ 1 * 9.8 - 9.0 * 9.5
$\begin{array}{c} 1 + 63 + 6^{3} + 9^{3} + a_{1} + s_{2} + a_{0} + s_{1} \\ - (-3) + 5 - 2^{-3} \\ + B + 5 \\ - 3 \\ - 3 \\ \end{array}$ we use the formula again to find S_2. $= a^{-4} + \beta^{-3} = -a_{1} + s_{2} - a_{0} + s_{2} 6 \\ - 129 \\ \text{Now we use the formula again to find S_2.}$ $= a^{-4} + \beta^{-3} = -a_{1} + s_{2} - a_{0} + s_{2} 6 \\ - 3 + 129 - 2^{-4} 65 \\ 37 - 130 \\ 57 \\ \text{we use the formula again to find S_2.}$ $= a^{-4} + \beta^{-9} = -a_{1} + s_{2} - a_{0} + s_{2} 6 \\ - 3 + 129 - 2^{-4} 65 \\ 77 - 130 \\ 57 \\ \text{we use the formula again to find S_2.}$ $= a^{-4} + \beta^{-9} = -a_{1} + s_{2} - a_{0} + s_{2} 6 \\ - 3 + 129 - 2^{-4} 65 \\ 77 - 120 \\ 77 \\ - 3^{3} + 27 - 2^{-4} 129 \\ - 3^{-4} 52^{-4} + 5^{-4} - 2^{-4} - 2^{-4} + 5^{-4} - 2^{-4} - 2^{-4} + 5^{-4} - 2^{-4} - 2^{-4} + 5^{-4} - 2^{-4} - 2^{-4} - 2^{-4} + 5^{-4} - 2^{-4} -$		Now we use the formula again to find S_3:	
$a_{2} = a_{1}^{-1} a_{2}^{-1} a$			-105 66
$= \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{2} + \frac{1}{9} + $		5_5=0~5+p~5=-a_1~5_2-a_0~5_1	= 133-00
$= \frac{1}{9}^{-3}$ Now we use the formula again to find \$.8: $= \alpha^{+9} + \beta^{+9} = -a_1^{++} \$_2^{-1} + a_0^{-+} \$_2^{-6} + \delta_2^{-6} $			= 129
$x w use the formula again to find $.8:= \alpha^{*8} + \beta^{*8} = -a_1^{*8} \cdot S_2^{*7} - a_10^{*8} \cdot S_2^{*6} + S_2^{*6} \cdot S_2^{*7} + S_2^{*6} + S_2^{*6} \cdot S_2^{*7} + S_2^{*6} \cdot S_2^{*7} + S_2^{*6} + S_2^{*7} + S_2^{*6} + S_2^{*7} + S_$			
w we use the formula again to find \$_10: Now we use the formula again to find \$_12: $0 = \alpha^{-10} + \beta^{-10} = -a_1^{+} \$_9^{-1} - a_0^{+} \$_8^{-1}$ Now we use the formula again to find \$_12: $339 \cdot 514$ $\$_12 = \alpha^{-1}2 + \beta^{-1}2 = -a_1^{+} \$_1^{-1} - a_0^{-} \$_1^{-1}0$ 25 $= (-c_3)^{+} 2049 + 2^{+} 1025$ w we use the formula again to find \$_11: $= 6147 - 2050$ $1 = \alpha^{-11} + \beta^{-11} = -a_1^{-1} \$_10 - a_0^{-1} \$_2^{-9}$ $= 4097$ $a_3) + 1025 - 2^{+}513$ So, the value of $\alpha^{-1}2 + \beta^{-1}2$ is 4097.		=9 Turner	Now we use the formula again to find \$_8:
$0 = \alpha^{-1}0^{+} \beta^{-1}0 = a_{-1}^{-1}s_{-9}^{-1}a_{-0}^{-1}s_{-8}^{-1}$ Now we use the formula again to find S_12: $(-3) \neq 513 - 2^{+}257$ S_12 = $\alpha^{-1}2 + \beta^{-1}2 = -a_{-1}^{+1}s_{-1}^{-1} - a_{-0}^{-1}s_{-1}^{-1}0$ 225 S_12 = $\alpha^{-1}2 + \beta^{-1}2 = -a_{-1}^{+1}s_{-1}^{-1} - a_{-0}^{-1}s_{-1}^{-1}0$ v we use the formula again to find S_11: $= (-3)^{+}2049 - 2^{+}1025$ v we use the formula again to find S_11: $= 6147 - 2050$ $1 = \alpha^{-1}1 + \beta^{-1}1 = -a_{-1}^{-1}s_{-1}^{-1}0 - a_{-0}^{-1}s_{-9}^{-1}$ $= 4097$ $-3) \neq 1025 - 2^{+}513$ So, the value of $\alpha^{-1}2 + \beta^{-1}2$ is 4097 .	v we us = a^8 -3) * 12 -37 - 13(57 - 13(= 9 = 9 = 9 = 0 = -a_1*S_7 - a_0*S_6 = -a_1*S_7 - a_0*S_6 = -a_1*S_7 - a_0*S_6 = -a_1*S_8 - a_0*S_7 = -a_1*S_8 - a_0*S_7 = -2*129 = -2*	Now we use the formula again to find \$_8:
$S_{33} + 514$ $S_{25} + 514$ $S_{25} + 514$ $S_{25} + 512 = \alpha^{12} + \beta^{12} = -a_1^{1*}S_{21} + a_0^{*}S_{21}^{-10}$ $= -(-3)^{*}2049 + 2^{*}1025$ $= -(-3$	v we us = a*8 -3) * 12 37 - 13(57 v we us = a*9 -3) * 25 71 - 25 3 3 v we us	= 9 = 9 T se the formula again to find S_8: + $\beta^{*8} = -a_1^{*} S_7^{*} - a_0^{*} S_6^{*} 6$ = 2^{*65} o se the formula again to find S_9: + $\beta^{*9} = -a_1^{*} S_8^{*} - a_0^{*} S_7^{*} 5$ $57 - 2^{*} 129$ S se the formula again to find S_10: = 0.0000	Now we use the formula again to find S_8:
Solution $S_112 = \alpha^{11}2 + \beta^{11}2 = -a_1^{1+1}S_1^{11} - a_0^{1+1}S_1^{10}$ γ_{225} $= -(-3)^{+}2049 + 2^{+}1025$ w we use the formula again to find S_11: $= 6147 - 2050$ $= -\alpha^{-11} + \beta^{-11} = -a_1^{-1} + S_1^{-10} - a_0^{-1} + S_2^{-10}$ $= 4097$ $-3)^{+1025 - 2^{+}513}$ So, the value of $\alpha^{-12} + \beta^{-12}$ is 4097.	v we us = a^8 = a^8 = a^9 = a^9 -3) * 25 771 - 256 13 3 v we us = a^9 -3) * 25 -3) * 25 -3 -3) * 25 -3 -3 -3 -3 -3 -3 -3 -3 -3 -3 -3 -3 -3	= 9 = 9 = 9 = 9 = 9 = 9 = 9 = 1*5.7 - a_0*5.6 = 9 - 2*65 0 se the formula again to find S_9: + β*9 = -a_1*5.8 - a_0*5.7 57 - 2*129 3 se the formula again to find S_10: 10 + β*10 = -a_1*5.9 - a_0*5.8 13 - 0*97	Now we use the formula again to find S_8:
$\begin{aligned} &= -(-3) * 2049 - 2 * 1025 \\ &= 6147 - 2050 \\ &= 6147 - 2050 \\ &= 4097 \\ -3) * 1025 - 2 * 513 \\ &&&\\ 275 - 1026 \end{aligned} \qquad $	v we us = a^8 37 - 13(37 37 37 - 13(37 - 13(- 3) * 25 - 3) * 31 - 3) * 31	= 9 = 9 T se the formula again to find S_8: + β ² 8 = -a_1*S_7 - a_0*S_6 89 - 2*65 0 se the formula again to find S_9: + β ² 9 = -a_1*S_8 - a_0*S_7 57 - 2*129 3 se the formula again to find S_10: 10 + β ² 10 = -a_1*S_9 - a_0*S_8 13 - 2*257 14	Now we use the formula again to find S_8:
w we use the formula again to find \$_11: 1 = α^11 + β^11 = -a_1 * \$_10 - a_0 * \$_9 -3) * 1025 - 2 * 513 075 - 1026 \$0, the value of α^12 + β^12 is 4097.	v we us = a*8 -3) * 12 37 - 13(57 -3) * 25 -3) * 25 -3) * 25 -3) * 51 -3) * 51 -3) * 51 -39 - 51	= 9 = 9 The set the formula again to find S_8: + $\beta^{*8} = -a_1^{*} S_2^{-1} - a_1^{*0} S_2^{-6}$ EVALUATE: $\beta^{*9} = -a_1^{*1} S_2^{-1} - a_1^{-0} S_2^{-6}$ Set the formula again to find S_9: + $\beta^{*9} = -a_1^{*1} S_2^{-1} - a_2^{-0} S_2^{-6}$ Set the formula again to find S_10: 10 + $\beta^{*10} = -a_1^{*1} S_2^{-9} - a_2^{-0} S_2^{-6}$ 13 - 2 * 257 14	Now we use the formula again to find S_8: Now we use the formula again to find S_12: $S_12 = \alpha^{1}2 + \beta^{1}2 = -a_1^{1}S_1^{1} - a_0^{S_10}$
1 = α ⁺ 11 = ⁺ a ₁ 1 * S ₂ 10 - a ₂ 0 * S ₂ 9 = 4097 -3) * 1025 - 2 * 513 075 - 1026 - So, the value of α ⁺ 12 + β ⁺ 12 is 4097.	v we us = a*8 -3) * 12 37 - 13(57 - 37 - 13(57 - 3) * 25 - 3) * 25 - 3) * 51 - 51 - 51 - 51 - 51 - 51 - 51 - 51 -	= 9^{-1} = 9^{-1} se the formula again to find S_8: + $\beta^{*8} = -a_1^{*} S_2^{-1} - a_1^{*0} S_2^{-6}$ EVALUATE: $9^{*2} = -a_1^{*1} S_2^{-1} - a_1^{-1} S_2^{-1}$ se the formula again to find S_9: + $\beta^{*9} = -a_1^{*1} S_2^{-1} - a_1^{-1} S_2^{-1}$ se the formula again to find S_10: 10 + $\beta^{*10} = -a_1^{*1} S_2^{-1} - a_2^{-1} S_2^{-1}$ 13 - $2^{*2} 2^{*2} 57$ 14	Now we use the formula again to find S_8: Now we use the formula again to find S_12: $S_12 = \alpha^{+}12 + \beta^{+}12 = -\alpha_1^{+}S_1^{+}1 - \alpha_2^{+}S_1^{-}10$ $= -(-3)^{+}2049 + 2^{+}1025$
-3) *1025 - 2 * 513 075 - 1026 - So, the value of α*12 + β*12 is 4097.	v we us = a*8 -3) * 12 37 - 13(57 v we us = a*9 -3) * 25 0 = a*1 -3) * 51 33 9 - 51 225 v we us	$= 9$ $= 9$ $= 9$ $= 9$ $= 9$ $= 9$ $= 9$ $= -a_1 + S_2 - a_0 + S_2 - 6$ $= 9 - 2^{+} 65$ $= 0$ $= 2^{+} 65$ $= -a_1 + S_2 - a_0 + S_2 - 7$ $= 5^{-} 2^{+} 129$ $= 8$ $= 8 \text{ the formula again to find S_10:}$ $= 0 + \beta^{-1} 0 = -a_1 + S_2 - a_0 - S_2 - 8$ $= 3 - 2^{+} 257$ $= 14$ $= 8 \text{ the formula again to find S_11:}$	Now we use the formula again to find S_8: Now we use the formula again to find S_12: $S_12 = \alpha^{-1}2 + \beta^{-1}2 = -a_1^{+}S_1^{-1} - a_0^{+}S_1^{-1}0$ $= -(-3)^{+}2049 + 2^{+}1025$ = 6147 - 2050
55 - 1026 - So, the value of α~12 + β~12 is 4097.	v we us = α^{*8} -3) * 12 37 - 130 57 v we us = α^{*9} -3) * 25 v we us -3) * 51 -39 - 51 25 v we us = α^{-11}	= 9 = 9 The set the formula again to find S_8: + $\beta^{*8} = -a_1^{*} S^{-7} - a^{0*} S^{-6}$ EVALUATE: $\beta^{*9} = -a_1^{*} S^{-8} - a^{0*} S^{-7}$ So the formula again to find S_10: EVALUATE: $0 + \beta^{*10} = -a_1^{**} S^{-9} - a^{0*} S^{-8}$ EVALUATE: $1 + \beta^{*11} = -a_1^{**} S^{-1} - a^{0*} S^{-9}$	Now we use the formula again to find S_8: Now we use the formula again to find S_12: $S_12 = \alpha^{-1}2 + \beta^{-1}2 = -a_1^{+}S_1^{-1} - a_0^{+}S_1^{-1}0$ $= -(-3)^{+}2049 + 2^{+}1025$ = 6147 - 2050 = 4097
	v we us = α*8 -3) * 12 37 - 130 57 -3) * 25 -3) * 25 -3) * 51 -3) * 51 -3) * 51 -39 - 51 -25 - v we us -3) * 51 -3) * 10 -3) * 10 -3) * 10	= 9 = 9 = 9 = 9 = 0 = 0 = 0 = 0 = 0 = 0 = 0 = 0	Now we use the formula again to find S_8: Now we use the formula again to find S_12: $S_12 = \alpha^{-1}2 + \beta^{-1}2 = -a_11^*S_11 - a_0^*S_10$ $= -(-3)^*2049 - 2^*1025$ = 6147 - 2050 = 4097

Figure 16: Chat GPT solution on If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{12} + \beta^{12}$ using Newton sum . Sk = $\alpha^{k} + \beta^{k} = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$.

2.2.12 If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{-13} + \beta^{-13}$ using Newton sum . Sk = $\alpha^{-k} + \beta^{-k} = -a_{-k-1} + S_{-k-1} - a_{-k-2} + S_{-k-2}$ is shown in **figure 17**

	If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{-13} + \beta^{-13}$ using	Now we use the formula again to find 5_4:
	Newton sum . Sk=ok+jk=-a1(bSk-1+cSk-2)	S_4=α^4+β^4=-a_1*S_3-a_0*S_2
		=-(-3)*9-2*5
a	To find the value of a^13 + R°13 using Newton's sums, we'll use the following formula-	= 277 - 10
.	To find the value of 0, 15 + p. 15 using reactor 5 surps, we'll use the following formula.	-17
	$Sk = \alpha^{+}k + \beta^{+}k = -a_{(1)} + S_{(k-1)} - a_{(2)} + S_{(k-2)}$	
	Given the equation $x^2 - 3x + 2 = 0$, the coefficients are:	Now we use the formula again to find S_5:
	a_2=1	
	a_1= -3	S_5=α^5+β^5=-a_1*S_4-a_0*S_3
		=-(-3) * 17 - 2 * 9
	Let's use Newton's sum to find of 3 + 6*13.	=51-18
		= 33
	2. S_1=α ^{x1} +β ^{x1} =-(-3)=3	Now we use the formula again to find S_6:
		S 6= c^6 + 6^6 = -a 1*S 5 - a 0*S 4
		=-(-3)*33-2*17
	$S_2 = \alpha^2 + \beta^2 = -a_1 + S_1 - a_0 + S_2 0$	-00.24
	=-(-3)^3-2*2	= 33-04
		= 6b
		Maurus use the formule equip to find 9.7.
	Now we use the formula again to find S_3:	Now we use the formula again to find 5_7.
	£ 3−m²2⊥B²3−.a 1#€ 9. a 0#€ 1	S_7=α*7+β*7=-a_1*S_6-a_0*S_5
	=-(-3)*5-2*3	=-(-3)*65-2*33
		= 195 - 66
	= Q There was an error generating a response	
Now	v we use the formula again to find S.8:	= 129
Now :S_8 = -(-	v we use the formula again to find S_8: = cr'8 + β^8 = -a_1*S_7-a_0*S_6 -3) * 129 - 2* 65 	= 129
Now S_8 = -(- = 38 = 29	we use the formula again to find S_8: = α^8 + β^8 = -a_1*S_7 - a_0*S_6 -3) *129 - 2* 65 37 - 130 57	= 129
Now S_8 = -(- = 38 = 29	v we use the formula again to find \$_8: = α^8 + β^8 = -a_1 * \$_7 - a_0 * \$_6 	= 129
Now S_8 =-(- = 38 = 29 Now	v we use the formula again to find S_8: = $\alpha^* 8 + \beta^* 8 = -a_1^* S_7^* - a_0^* S_6^* = -a_1^* S_7^* = -a_1^* S_$	= 129
Now 5_8 =-(- =36 =25 Now 5_9 (-	we use the formula again to find §_8: = $\alpha^* 8 + \beta^* 8 = -a_1 1^* S_7 - a_0 0^* S_6$ -3) * 129 - 2* 65 37 - 130 57 we use the formula again to find §_9: = $\alpha^* 9 + \beta^* 9 = -a_1 1^* S_8 8 - a_0 0^* S_7$	= 129
Now S_8 =-(- =38 =25 Now S_9 =-(- =77	we use the formula again to find §_8: = $\alpha^* 8 + \beta^* 8 = -a_1^* \$_7^- a_0^* \$_6^- 6$ -3) * 129 - 2* 65 37 - 130 57 we use the formula again to find §_9: = $\alpha^* 9 + \beta^* 9 = -a_1^* \$_8 - a_0^* \$_7^- 7$ -3) * 257 - 2* 129 7 - 258	= 129
Now 5_8 = -(- = 38 = 25 Now 5_9 = -(- = 77 = 51	v we use the formula again to find S_8: = $\alpha^*8 + \beta^*3 = -a_1^*S_7^-a_0^*S_6^-633 + 129 - 2^*65371305713057575757575757575757$	= 129
Now S_8 = -{- = 38 = 29 = -{- = 77 = 51 Now	v we use the formula again to find \$_8: = $\alpha^*8 + \beta^*8 = -a_1^* \$_7^- a_0^* \$_6^- 6$ = $\alpha^*8 + \beta^*8 = -a_1^* \$_7^- a_0^* \$_6^- 6$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_7^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_8^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_8^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_8^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \$_8^- a_0^* \$_8^- 7$ = $\alpha^*9 + \beta^*9 = -a_1^* \$_8^- a_0^* \bullet_8^- a_0^* \$_8^- a_0^* \$_8^- a_0^* \ast_8^- a_0^* \bullet_8^- a_0^* \ast_8^- a_0^* \bullet_8^- a_0^- a_0^* \bullet_8^- a_0^- a_0^* \bullet_$	= 129 Near we use the formula again to find 8,10-
Now S_8 = -{- = 38 = 25 Now S_9 = -{- = 77 = 51 Now S_10	we use the formula again to find \$_8: = $\alpha^*8 + \beta^*8 = -a_1^* \$_2^* - a_0^* \$_2^* 6$ $33^* 129 - 2^* 65$ 37 - 130 57 we use the formula again to find \$_9: = $\alpha^*9 + \beta^*9 = -a_1^* \$_3^* - a_0^* \$_2^* 7$ $33^* 257 - 2^* 129$ 71 - 258 3 w we use the formula again to find $\$_2^* 10$: $0 = \alpha^*10 + \beta^*10 = -a_1^* \$_2^9 - a_0^* \$_2^* 8$	= 129 Now we use the formula again to find S, 12- S, 12 = u:12 + β'12 = -u, 1 * S, 11 - u, 0 * S, 10
Now S_8 =-{- =-38 =-29 =-{- =-77 =-51 Now S_10 =-{-	$\label{eq:second} \begin{array}{l} & & & & & & & & \\ & = \alpha^*8 + \beta^*8 = -a_11^* \mathbb{S}_2^* - a_2 0^* \mathbb{S}_2^* 6 \\ & = \alpha^*8 + \beta^*8 = -a_11^* \mathbb{S}_2^* - a_2 0^* \mathbb{S}_2^* 6 \\ & = \alpha^*9 + \beta^*9 = -a_11^* \mathbb{S}_2^* - a_2 0^* \mathbb{S}_2^* 7 \\ & & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & & \\ & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\$	Nraw we use the formula again to find S_{-1} ?- S_{-1} 2 = u_{-1} 2 + β -12 = u_{-1} 1 + δ_{-1} 7 - = -(-3) + 2049 - 2 + 1025
Now <u>S_8</u> =-{- =-38 =-28 <u>S_9</u> =-{- =-77 =51 Now <u>S_10</u> =-{- =15	$x we use the formula again to find $_8:$ $= \alpha^* 8 + \beta^* 8 = -a_1^* \$_7^- a_0^* \$_6$ $33^* 129 - 2^* 65$ $37 - 130$ 57 $w we use the formula again to find $_9:$ $= \alpha^* 9 + \beta^* 9 = -a_1^* \$_\$_8 - a_0^* \$_7$ $33^* 257 - 2^* 129$ $71 - 258$ 3 $w we use the formula again to find $_10:$ $D = \alpha^* 10 + \beta^* 10 = -a_1^* \$_9 - a_0^* \$_8$ $-3)^* 513 - 2^* 257$ $39 - 514$	Now we use the formula again to find 8,10- $8,12 = u(12 + \beta^{-1}2 = u_{1}^{-1}8, 11 + u_{1}^{0} + 8,10)$ $= -(-3) + 2049 \cdot 2 + 1025$ $- 6447 \cdot 2050$
Now S_8 = -{- = 3& = 2 & S_9 = -{- = 51 Now S_10 = -{- = 15 = 10	wwe use the formula again to find \$_8: = $\alpha^*8 + \beta^*3 = -\alpha_1^* \$_2^* - \alpha_2^* \$_2^* = 0$ $33^* 129 - 2^* 65^* = 37^* - 32^* - 3$	= 129 Now we use the formula again to find 5,10- 5,12 = u-12 + p+12 = xu,1 + 5,-11 - u,0 + 5,10 = -(-3) * 2049 - 2 * 1025 = 6147 2050 = 4057
Now 5.8 = = 38 = -2 = -2 = 51 = 51 Now S_10 = = 15 = 10 Now	we use the formula again to find S_8: = $\alpha^* 8 + \beta^* 8 = -a_1^* S_7^* - a_0^* S_6^* 6$ = $\alpha^* 8 + \beta^* 8 = -a_1^* S_7^* - a_0^* S_6^* 6$ = $\alpha^* 9 + \beta^* 9 = -a_1^* S_8^* - a_0^* S_7^* 7$ = $\alpha^* 9 + \beta^* 9 = -a_1^* S_8^* - a_0^* S_7^* 7$ = $\alpha^* 9 + \beta^* 9 = -a_1^* S_8^* - a_0^* S_7^* 7$ = $\alpha^* 9 + \beta^* 9 = -a_1^* S_8^* - a_0^* S_7^* 7$ = $\alpha^* 9 + \beta^* 10 = -a_1^* S_9^* - a_0^* S_8^* 8$ = $\alpha^* 10 + \beta^* 10 = -a_1^* S_9^* - a_0^* S_8^* 8$	Now we use the formula again to find $\$_1$?- $\$_1$?= a_1 ? \bullet_1 ?= a_1 ? \bullet_2 ?= a_2 ? \bullet_2 ? \bullet_3 ? \bullet_3 ? \bullet_4 $= (-3)^4 2040 \cdot 2^{-1}1025$ $= (-6147 \cdot 2050)$ = 4057 Now we use the formula again to find $\$_1$? \bullet_3 ? Now we use the formula again to find $\$_1$? \bullet_3 ?
Now S_8 = -{- = 3% = 2! Now S_9 = -{- = 777 = 51 Now S_10 Now S_10 Now	ww use the formula again to find S_8: = $\alpha^* 8 + \beta^* 8 = -a_1 1^* S_1^* 7 - a_0 0^* S_1^* 6$ = $\alpha^* 8 + \beta^* 8 = -a_1 1^* S_1^* 7 - a_0 0^* S_1^* 6$ = $\alpha^* 9 + \beta^* 9 = -a_1 1^* S_1 8 - a_0 0^* S_1^* 7$ = $\alpha^* 9 + \beta^* 9 = -a_1 1^* S_1 8 - a_0 0^* S_1^* 7$ = $\alpha^* 10 + \beta^* 10 = -a_1 1^* S_1 9 - a_0 0^* S_1^* 8$ = $\alpha^* 11 + \beta^* 11 = -a_1 1^* S_1 0 - a_0 0^* S_1 9$	Now we use the formula again to find $8, 12^{-1}$ $8, 12 = a^{-1}12 + \beta^{-1}12 = -a_{-1}1 + 8, -11 - a_{-0}0 + 8, 10$ $= -(-3)^{+}2049 + 2^{-1}1026$ = -6447 + 2050 = 4087 Now we use the formula again to find $8, 13^{-1}$ $8, 13 = a^{-1}18 + 19^{-1}18 = -a_{-1}1^{-1}8, 12^{-1}a_{-1}0^{-1}8, 11$ $= -(-3)^{+}4087 + 19^{-1}38 = -a_{-1}1^{-1}8, 12^{-1}a_{-1}0^{-1}8, 11$ $= -(-3)^{+}4087 + 9^{-1}20 = -a_{-1}1^{-1}8, 12^{-1}a_{-1}0^{-1}8, 11$
Now S_8 = -{- = 38 = 28 S_9 = -{- = 51 Now S_10 = -{- = 15 = 10 Now S_111 = -{-	ww use the formula again to find S_8: = $\alpha^* 8 + \beta^* 8 = -a_1 1^* S_1^* 7 - a_2 0^* S_2^* 6$ $3) * 129 - 2^* 65$ 37 - 130 57 ww use the formula again to find S_9: = $\alpha^* 9 + \beta^* 9 = -a_1 1^* S_1^* 8 - a_2 0^* S_2^* 7$ $33 * 257 - 2^* 129$ $1^* - 258$ 3 ww use the formula again to find S_10: $0 = \alpha^* 10 + \beta^* 10 = -a_1 1^* S_2 9 - a_2 0^* S_2^* 8$ $33 * 513 - 2^* 257$ 39 - 514 225 ww use the formula again to find S_11: $1 = \alpha^* 11 + \beta^* 11 = -a_1 1^* S_1 10 - a_2 0^* S_2 9$ $3) * 1025 - 2^* 513$	Now we use the formula again to find $8, 12^{-1}$ $8, 12 = a^{-1}12 + \beta^{-1}12 = a^{-1}1 + a^{-0}1 + a^{$
Now S_8 = -{- = 38 = 25 Now S_9 = -{- = 51 Now S_10 = -{- = 15 = 10 Now S_11 = -{- = -{- = 30	ww use the formula again to find \$_8: = $\alpha^* 8 + \beta^* 8 = -a_1 1^* \$_7 7 - a_0 0^* \$_6 6$ = $\alpha^* 8 + \beta^* 8 = -a_1 1^* \$_7 7 - a_0 0^* \$_6 6$ = $\alpha^* 9 + \beta^* 9 = -a_1 1^* \$_8 - a_0 0^* \$_7 7$ = $\alpha^* 9 + \beta^* 9 = -a_1 1^* \$_8 - a_0 0^* \$_7 7$ = $\alpha^* 9 + \beta^* 9 = -a_1 1^* \$_8 - a_0 0^* \$_7 7$ = $\alpha^* 9 + \beta^* 9 = -a_1 1^* \$_8 - a_0 0^* \$_7 7$ = $\alpha^* 10 + \beta^* 10 = -a_1 1^* \$_9 - a_0 0^* \$_8 8$ = $\alpha^* 11 + \beta^* 11 = -a_1 1^* \$_1 10 - a_0 0^* \$_9 9$ = $\alpha^* 11 + \beta^* 11 = -a_1 1^* \$_1 10 - a_0 0^* \$_9 9$ = $\alpha^* 11 + \beta^* 11 = -a_1 1^* \$_1 10 - a_0 0^* \$_9 9$	Now we use the formula again to find 8,17- $S_1T2 = u^{1}T2 + \beta^{-1}T2 = -u_11^{-3}.71 - u_20^{-3}.710$ $= -(-5)^{-4} 20.49 - 2^{-1} 10.25$ - 6147 - 2050 = 40877 Now we use the formula again to find 8,13: $S_1T3 = aT13 + \beta^{-T}3 = -u_11^{-3}.712 - u_20^{-3}.711$ $= -(-5)^{-4} 40.877 - 9^{-9}.90.49$ = 12291 - 40.981 = 21291 - 40.981 = 2193

Figure 17: Chat GPT solution on If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{13} + \beta^{13}$

2.2.13 If the roots of the equation equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find the value of $\alpha \wedge 14 + \beta \wedge 14$ using Newton sum. Sk = $\alpha \wedge k + \beta \wedge k = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$ is shown in figure 18



Now we use the formula again to find S_12:
S_12 = α^12 + β^12 = -a_1 * S_11 - a_0 * S_10
=-(-3)*2049-2*1025
= 6147 - 2050
= 4097
Now we use the formula again to find S_13 :
S_13=α^13+β^13=-a_1*S_12-a_0*S_11
=-(-3) * 4097 - 2 * 2049
= 12291 - 4098
= 8193
Now we use the formula again to find S_14 :
S_14 = α^14 + β^14 = -a_1 * S_13 - a_0 * S_12
= -(-3) * 8193 - 2 * 4097
= 24579 - 8194
So, the value of $\alpha^{*}14 + \beta^{*}14$ is 16385.

Figure 18: Chat GPT solution on If the roots of the equation equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{14} + \beta^{14}$

2.2.14 If the roots of the equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find the value of $\alpha \wedge 15 + \beta \wedge 15$ using Newton sum . Sk = $\alpha \wedge k + \beta \wedge k = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$ is shown in figure 19

If the roots of the equation $x^2 - 3x + 2 = 0$ are α and β , find the value of $\alpha^{-15} + \beta^{-15}$ using Newton sum. Sk= α k+ β k= $-a1$ (bSk- $-1+c$ Sk- -2)	Now we use the formula again to find S_4:
	$S_4 = \alpha^4 + \beta^4 = -a_1 \cdot S_3 - a_0 \cdot S_2$
	=-(-3)*9-2*5
${}_{50}^{60}$ To find the value of α^{*15} + β^{*15} using Newton's sums, we'll use the following formula:	(=27-10
$Sk = \alpha^{*}k + \beta^{*}k = -a_{-}(1) * S_{-}(k-1) - a_{-}(2) * S_{-}(k-2)$	= 17
Given the equation $x^2 - 3x + 2 = 0$, the coefficients are:	Now we use the formula again to find S_5:
a.2=1	
a_1=-3	$S_5 = \alpha^5 + \beta^5 = -a_1 + S_4 - a_0 + S_3$
a_0=2	=-{-3}*17-2*9
	= 51 - 18
Let's use Newton's sum to find $\alpha^{-15} + \beta^{-15}$:	=33
1. S_0 = α^0 + β^0 = 2	
2. S_1=α^1+β^1=-(-3)=3	Now we use the formula again to find S_6:
Now we can use the formula to find \$_2:	S_6=α^6+β^6=-a_1*S_5-a_0*S_4
S 2= a*2 + 6*2 = -a 1*S 1 - a 0*S 0	=-(-3)*33-2*17
=-(-3)*3-2*2	= 99 - 34
=9-4	= 65
= 5	
	Now we use the formula again to find S_7:
Now we use the formula again to find \$_3:	
S 3=m ³ +m ³ =-a 1*S 2-a 0*S 1	$S_1 = \alpha 7 + \beta^2 7 = -\alpha_1 * S_6 - \alpha_0 * S_5$
=-(-3)*5-2*3	=-(-3)*65-2*33
= 15 - 6	= 195 - 66
= 9	= 129



Figure 19: Chat GPT solution on If the roots of the equation $x^2 - 3x + 2 \equiv 0$ are α and β , find the value of $\alpha^{15} + \beta^{15}$

2.2. 14 If the roots of the equation $x \wedge 2 - 3x + 2 = 0$ are α and β , find the value of $\alpha \wedge 28 + \beta \wedge 28$ using Newton sum . Sk = $\alpha \wedge k + \beta \wedge k = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$ is shown in figure 20

```
FE If the roots of the equation x2 - 3x + 2 = 0 are \alpha and \beta, find the value of \alpha^2 28 + \beta^2 28

To find the value of \alpha^{28} + \beta^{28}, we can continue using Vieta's formulas and Newton's Sums.

Given the quadratic equation x^2 - 3x + 2 = 0 with roots \alpha and \beta, we have the following values from Vieta's formulas:

1. Sum of roots: \alpha + \beta = 3

2. Product of roots: \alpha\beta = 2

We can use Newton's Sums to find the value of \alpha^{28} + \beta^{28}. The formula for S_k with k = 28

is:

S_{28} = \alpha^{28} + \beta^{28} = -\frac{1}{a} (bS_{27} + cS_{26})
```

We can calculate the values of S_{27} and S_{26} using the previously calculated values: 1. $S_{27} = \alpha^{27} + \beta^{27}$. Unfortunately, we don't have a formula for S_{27} at this point, but we can still find it using the relationship between S_k and S_{k-1} derived from Newton's Sums: $S_k = -\frac{1}{a} (bS_{k-1} + cS_{k-2})$ So, we can start by calculating S_{27} and then use it to find S_{28} . Let's first calculate S_{27} : $S_{27} = -\frac{1}{1} (-3S_{26} + 2S_{25})$ We'll need to calculate S_{28} and S_{25} using the same recursive formula. Continuing this process, we'll eventually calculate S_{27} , and then we can calculate S_{28} using the formula above. However, this recursive process can become quite complex and time-consuming for higher values of k. If you're looking for a more efficient method, you might consider using a mathematical software or programming language that can handle symbolic computations.

Figure 20: Chat GPT solution on If the roots of the equation $\mathbf{x} \wedge \mathbf{2} - 3\mathbf{x} + 2 = 0$ are α and β , find the value of $\alpha \wedge \mathbf{28} + \beta \wedge \mathbf{28}$ using Newton sum . Sk = $\alpha \wedge k + \beta \wedge k = -a_{(k-1)} * S_{(k-1)} - a_{(k-2)} * S_{(k-2)}$

3. Results:

The results of this study provide concrete evidence of Chat GPT's superior utilization of the Newton Sum Method for calculating quadratic root functions. The model adeptly computes expressions of the form $\alpha ^{15} + \beta ^{15}$ using this method, while its proficiency using algebraic formula methods is generally limited to $\alpha ^{14} + \beta ^{15}$ 4. This striking discrepancy underscores the transformative impact that different methods can have on elevating the AI system's mathematical provess.

In the realm of quadratic root function calculations, the Newton Sum Method emerges as a powerful tool compare Algebra Formula method, enabling AI systems like $\alpha \wedge 15 + \beta \wedge 15$ which can not be solve by Algebra formula method. However, a noteworthy facet emerges as ChatGPT encounters challenges in yielding definitive solutions for expressions such as $\alpha \wedge 28 + \beta^{28}$. The intricate nature of these solutions proves to be a stumbling block, leading to limitations in the scope of deliverable conclusions.

The core reason behind these limitations lies in the inherent methodology of the Newton Sum calculation technique. Rooted in historical calculations, this method relies on previous results to determine the current outcome. As a consequence, when confronted with higher-order quadratic root function calculations like α * 28 + β * 28. the computational process undergoes a transformation into a labyrinthine endeavour, entailing increased intricacy and time consumption.

Engaging with this limitation, Chat GPT offers a comprehensive strategy: "We'll need to calculate S26 and S25 us-

ing the same recursive formula. Continuing this process, we'll eventually calculate S27, and then we can calculate S28 using the formula above. However, this recursive process can become quite complex and time-consuming for higher values of k. If you're looking for a more efficient method, you might consider using a mathematical software or programming language that can handle symbolic computations."

4. Conclusion

Pushing Boundaries: Pioneering Novel Maths Approaches for Overcoming Limitations in AI.

This study serves as an illuminating testament to the significance of pioneering innovative methodologies, rules, theorems, or formulas to surmount the current limitations in AI systems like Chat GPT. These innovative pursuits hold the key to unlocking the untapped potential that lies within, propelling AI systems to greater heights of proficiency. In essence, they offer a strategic pathway towards expanding the capabilities of AI and pushing the boundaries of what can be achieved.

In the broader panorama of AI's evolutionary trajectory, these findings resonate as a resonant call to action. They invite researchers, developers, and practitioners to seize the opportunity and embark on a transformative journey. The limitations encountered today are not mere roadblocks but stepping stones towards a more capable AI landscape. As the digital realm continues to evolve, the pursuit of novel methodologies becomes an integral part of this evolutionary symphony.

These findings cast a compelling vision for future research endeavours. They beckon researchers to embark on audacious explorations into uncharted territories. This involves delving deep into the unexplored realm of novel mathematical techniques, intricately tailored to accommodate the unique characteristics of AI systems. Such an endeavour, though challenging, promises to be transformative. The resulting synergy between innovative mathematics and advanced AI systems holds the potential to reshape the technological landscape.

In conclusion, this study resonates as a clarion call to innovation, urging stakeholders to embrace the challenge and forge ahead. The limitations encountered today are the building blocks of progress. The pioneering of novel methodologies is not only an aspiration but a necessity to shape the future of AI. As we stand at the precipice of an AI-powered future, the beckoning of uncharted territories invites us to create a new paradigm, where the potential of AI systems is truly unleashed and their impact resonates across the technological landscape.

5. Discussion

The results obtained from this study strongly emphasize the substantial impact that the choice of method holds in bolstering the mathematical proficiencies of ChatGPT. One method that particularly stands out is the application of the Newton Sum Method, which emerges as a compelling and illustrative example.

This method acts as a pivotal gateway, enabling the model to transcend its previous limitations and boldly explore the domain of calculations involving higher exponents. This significant breakthrough not only showcases the Newton Sum Method's efficacy but also highlights its pivotal role in pushing the boundaries of ChatGPT's mathematical capabilities.

The potential reason is that a simple solution can overcome the limitations of AI systems like Chat GPT in mathematical problem-solving. Simplified solutions can effectively address complex issues and make problem-solving more accessible and reliable. This is in line with Albert Einstein's quotation: Everything should be made as simple as possible.

In addition, Albert Einstein's also quote:

We cannot solve our problems with the same thinking we used when we created them .

ii) If you can't explain it simply you don't understand it well enough,

iii)"Genius is making complex ideas simple, not making simple ideas complex."

iv). "Any intelligent fool can make things bigger and more complex. It takes a touch of genius - and a lot of courage - to move in the opposite direction."

v)God always takes the simplest way.

vi)When the solution is simple, God is answering.

Isaac Newton quote Nature is pleased with simplicity. And nature is no dummy.

From the Albert Einstein's and Isaac Newton quote above, it can be seen that simplifying knowledge is very important.

Future Research:

The discoveries unearthed by this study carry a dual impact, contributing not solely to the advancement of AI's mathematical competencies but also underscoring the imperative of forging ahead with innovation. These findings serve as a clarion call for the exploration and creation of pioneering methodologies, novel rules such as Peter Chew Rule, inventive theorems, and ground breaking formulas. The aim is clear: to propel AI systems like Chat GPT to even greater heights of proficiency and utility.

Looking ahead, the vista of future research beckons. A promising avenue lies in the exploration and refinement of novel mathematical techniques that are expressly tailored to the unique demands of AI systems. This deliberate alignment holds the potential to significantly expand the purview of AI's capabilities, allowing them to permeate a diverse spectrum of problem-solving domains.

The major contributions of these findings not only contribute to AI's mathematical competencies but also emphasize the need for pioneering new methods, rules Such as Peter Chew Rule, theorems, or formulas to further enhance AI systems like Chat GPT and integrating domain-specific knowledge into the system to improve Chat GPT's mathematical competence. Future research could explore the development of novel mathematical techniques tailored to AI systems, thus expanding their capabilities across diverse problem-solving domains

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