

ARTICLE

## Optimized Cauchy-Gaussian Blend Model for Stochastic-Parametric Simulation of Seismic Ground Motions

R. Sharbati<sup>1\*</sup> H. R. Amindavar<sup>2\*</sup> H. R. Ramazi<sup>3</sup> S. Foti<sup>4</sup> B. Farzanegan<sup>2</sup>

1. Department of Civil and Environmental Engineering, Amirkabir University of Technology, Tehran, Iran

2. Department of Electrical Engineering, Amirkabir University of Technology, Tehran, Iran

3. Department of Mining & Metallurgical Engineering Amirkabir University of Technology, Tehran, Iran

4. Department of Structural, Geotechnical and Building Engineering, Politecnico di Torino, Torino, Italy

**Received:** 10 May 2021; **Accepted:** 30 May 2022; **Published Online:** 2 June 2022

**Abstract:** This article proposes a stochastic model for generation of synthetic seismic ground motions. In the first step, the wavelet coefficients of a record are extracted by the dual-tree complex discrete wavelet transform (DT-CDWT) and then they are simulated by an optimized Cauchy-Gaussian blend (CGB) model. This model predicts well the energy distribution of seismic ground motions, because in this model, the Gaussian distribution simulates smooth peaks and the Cauchy distribution is used to simulate impulsive peaks. Also, this model simulates several ascending-descending cycles in the time domain, predicts multiple frequency peaks each time, and simulates sequence-type records.

**Keywords:** Seismic ground motions; Cauchy-Gaussian blend model; Complex discrete wavelet transform; Genetic algorithm; Spectral and temporal nonstationarity

### 1. Introduction

The ground motions (GMs) that are recorded from past earthquakes, are very valuable, because they contain important information about the source and mechanism of earthquake occurrence. Also, the GMs of strong earthquakes are important for design engineers, because these GMs caused severe damage to existing structures.

For this reason, in areas with high or moderate seismic hazard, recorded GMs are used for dynamic analysis and design of structures to mitigate seismic risk. The GMs that will be used for the design of a structure, should be in accordance with the site where that structure will be built in the future. These GMs are rare or they do not exist for some source and site conditions. It is also proved that

\*Corresponding Author:

Reza Sharbati,

Department of Civil and Environmental Engineering, Amirkabir University of Technology, Tehran, Iran;

Email: [r.sharbati@aut.ac.ir](mailto:r.sharbati@aut.ac.ir); [re.sharbati@gmail.com](mailto:re.sharbati@gmail.com)

Hamidreza Amindavar,

Department of Electrical Engineering, Amirkabir University of Technology, Tehran, Iran;

Email: [hamidami@aut.ac.ir](mailto:hamidami@aut.ac.ir); [hamindavar@gmail.com](mailto:hamindavar@gmail.com)

DOI: <https://doi.org/10.54963/ptnd.v1i1.62>

Copyright © 2022 by the author(s). Published by UK Scientific Publishing Limited. This is an open access article under the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

the use of scaled GMs from other sites is unrealistic <sup>[1-3]</sup>. Hence, it's inevitable to use stochastic models that are able to generate realistic synthetic GMs.

In site-based models, the GMs recorded during previous earthquakes are used to extract a model for the simulation of realistic synthetic GMs for a site. These models are attractive because recorded GMs are available to seismologists. These models are less time-consuming than source-based models, and only require available source and site information <sup>[4]</sup>. The site-based stochastic models should be able to simulate the spectral and temporal nonstationary characteristics of real records. This is because under seismic excitation, the response of structures depends on their nonstationary characteristics. These models should also be parametric and have as less parameters as possible to generate synthetic records for the future events. For example, the models proposed by Mobarakeh et al. <sup>[5]</sup>, Wen and Gu <sup>[6]</sup>, Zhang and Zhao <sup>[7]</sup>, Zhang et al. <sup>[8]</sup>, and Wang et al. <sup>[9]</sup> are aimed to generate synthetic GMs with the same nonstationary characteristics with the recorded ones, but these models are nonparametric or contain many parameters. This model was recently by Sabetta and Pugliese <sup>[10]</sup>, and Stanford et al. <sup>[11]</sup>, their parameters are estimated for the seismic events, but these models underestimate the uncertainty of real GMs and neglect their temporal nonstationary characteristics.

Rezaeian and Der Kiureghian <sup>[4,12,13]</sup> proposed a site-based model that has few parameters and considers all sources of uncertainty. This model demands high computational effort and cannot simulate multiple frequency peaks. The Rezaeian's model <sup>[4]</sup> was later used by Medel-Vera and Ji <sup>[14]</sup> to generate synthetic records for the European areas. This model was recently modified by Tsioulou et al. <sup>[15]</sup> and Vetter et al. <sup>[16]</sup> to improve its capabilities. In the model proposed by Yamamoto and Baker <sup>[17]</sup>, the wavelet packet transform, the exponential distribution, and the bivariate lognormal distribution are used to generate synthetic records. In a follow-up research, Huang and Wang <sup>[18]</sup> have developed spatial correlation models for the parameters of Yamamoto's model <sup>[17]</sup> to extend it to regional-scale applications. Vlachos et al. <sup>[19,20]</sup> have developed a nonstationary version of the Kanai-Tajimi model for the simulation of frequency-time distribution of real records. Recently, Sharbati et al. <sup>[21]</sup> have developed a model based on the complex wavelet transform and mixture distributions. Three other site-based models recently developed for the simulation of seismic records, are expressed in Dak Hazirbaba and Tezcan <sup>[22]</sup>, Tezcan et al. <sup>[23]</sup>, and Wang et al. <sup>[24]</sup>. Also, several models were proposed to estimate well the temporal and spectral nonstationary characteristics of earthquake ground motions. These meth-

ods overcome some shortcomings of the previous models.

Most of the recorded GMs have complex structures. Hence simulation of these GMs by the stochastic-parametric models is controversial. The previous models are unable to simulate sharp and impulsive peaks, estimate accurately the frequency-time distribution of records, and simulate the GMs that have several ascending-descending cycles in the amplitude or multiple frequency peaks in a time. A model based on the dual-tree complex discrete wavelet transform (DT-CDWT) and Cauchy-Gaussian blend (CGB) distribution is proposed to overcome these shortcomings. By using the CGB model, sharp and impulsive peaks are simulated by the Cauchy distribution and smooth peaks by the Gaussian one. The simulation results are compared with the target records as well as the results of the Sharbati's model <sup>[21]</sup> and the Vlachos's model <sup>[19]</sup>. Here, the model proposed by Sharbati et al. <sup>[21]</sup> is named as the GB model. The mathematical basis of CGB model is presented in the next section.

## 2. Methodology

In the proposed model, the DT-CDWT extracts wavelet coefficients (WCs) of a record and the CGB distribution simulates the square of WCs. These two approaches are explained in more detail in this section.

### 2.1 Dual-tree Complex Discrete Wavelet Transform

The ordinary discrete wavelet transform (DWT) suffers from some problems such as lack of shift invariance, insufficient accuracy in the feature extraction, and lack of directional selectivity. These shortcomings are more obvious when modeling the frequency-time distribution of GMs. By using DT-CDWT, these shortcomings of DWT are improved. The block diagram of DT-CDWT, shown in Figure 1, is expressed in Sharbati et al. <sup>[21]</sup>. The filter bank coefficients of Kingsbury <sup>[25]</sup> are also used for the analysis of records.

### 2.2 Cauchy-Gaussian Blend Model

Modeling and simulation of GMs has been one of the major topics in earthquake engineering. The spectral and temporal nonstationarity of GMs should be simulated by robust stochastic models. In order to develop a parametric model for the simulation of records, their temporal amplitude should be simulated by an appropriate Probability Density Function (PDF). In practice, the most popular distribution used by earthquake engineers is the Gaussian distribution but this distribution cannot estimate fat tails and sharp peaks of GMs. We can use the Cauchy distribution to overcome these shortcomings, but it leads

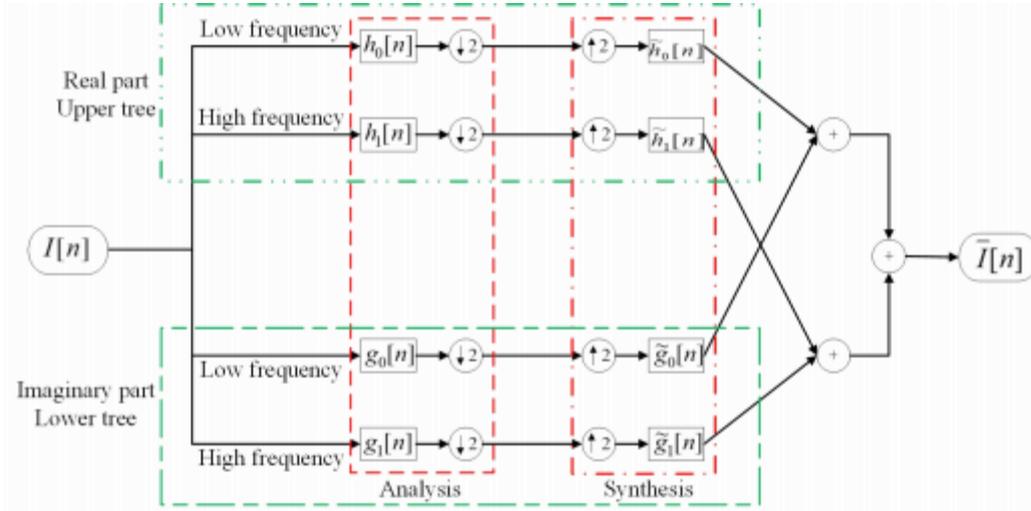


Figure 1. The diagram of one-level DT-CDWT.

to over-protection in descending phase of GMs. Because of the complex structure of GMs, it is realized that the Gaussian and Cauchy distributions are not appropriate for estimating the temporal amplitude of GMs, because the Gaussian distribution underestimates sharp peaks of GMs while the Cauchy distribution often underestimates their smooth peaks.

Among solutions to tackle these problems, we can use the Gaussian blend (GB) model, which approximates the PDF by a finite number of Gaussian distributions, but this model cannot predict sharp peaks and heavy-tailed PDFs. Another solution is to fit a Cauchy-Gaussian blend (CGB) distribution to a PDF. This mixture distribution controls the balance between the Gaussian and the heavy-tailed Cauchy. The relations expressing the Cauchy and Gaussian distributions are given in Equation (1) and Equation (2) respectively:

$$p_c(x) = \frac{1}{\pi\gamma \left[1 + \left(\frac{x-x_0}{\gamma}\right)^2\right]} = \frac{1}{\pi\gamma} \left[\frac{\gamma^2}{(x-x_0)^2 + \gamma^2}\right] \quad (1)$$

$$p_n(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (2)$$

where  $x_0$  and  $\gamma$  are the location and scale of the Cauchy distribution,  $\mu$  and  $\sigma$  are the location and variance of the Gaussian distribution. Suppose that we want to use a CGB model constructed by a Gaussian distribution and a Cauchy distribution for simulation of data. The mixture model is defined as follows:

$$p(x) = \alpha p_n(x) + (1 - \alpha) p_c(x) \quad (3)$$

Equation (3) has five parameters including two parameters for the Cauchy distribution ( $x_0, \gamma$ ), two parameters

for the Gaussian distribution ( $\mu, \sigma$ ) and a weighting coefficient ( $\alpha$ ). In order to fit a mixture of Cauchy and Gaussian PDFs to databased on the observation of  $N$  samples,  $\{x_1, x_2, \dots, x_N\}$ , we should estimate the unknown parameters of Cauchy and Gaussian PDFs in the characteristic function domain, as follows:

$$\phi(\omega) = E\{e^{j\omega x}\} = \alpha \exp(j\mu\omega - 0.5\sigma^2\omega^2) + (1 - \alpha) \exp(jx_0\omega - \gamma|\omega|) \quad (4)$$

$$\hat{\phi}(\omega) = \frac{1}{N} \sum_{n=1}^N e^{j\omega x_n} \quad (5)$$

where  $\omega$  is circular frequency and  $N$  is the number of actual data. Now, we want to minimize following cost function to obtain parameter values that minimize differences between the estimation provided by the CGB model and the actual data:

$$J = \int_{-\infty}^{+\infty} \phi(\omega) - \hat{\phi}(\omega) \Big| e^{-b^2\omega^2} d\omega \quad (6)$$

where  $b$  is a constant parameter that should be optimized for the application of interest. To minimize the above cost function and extract appropriate values for the CGB model's parameters, we have used the genetic algorithm. Genetic algorithm (GA) is a biologically inspired algorithm mimicking the genetic process proposed by Holland [26], belonging to Evolutionary Algorithms (EA) [27]. It is a popular method for global optimization, imitating the evolution of the living beings explained by Charles Darwin. The achieved solution after applying GA is an optimal solution to the minimization problem in Equation (6).

Based on the above model, a stochastic-parametric model is developed for the simulation of GMs that have sharp and impulsive peaks in their time domain, or those

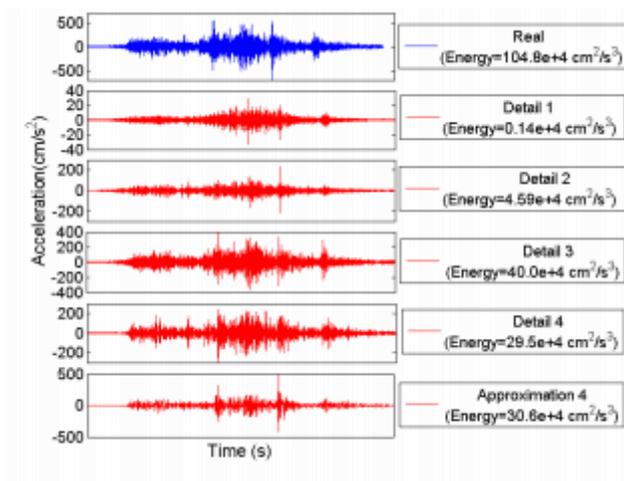
that exhibit fatter tails than typical GMs. The previous models underestimate the spectral and temporal characteristics of these GMs. The fundamental steps of the proposed method are explained in the next section.

### 3. Proposed Model

In the CGB model, it is required to follow three steps: 1) applying the DT-CDWT and extracting the WCs of a record, 2) simulating the coefficients of each level by the CGB model and extracting the parameters of the model by GA, and 3) generating synthetic WCs for each level and applying the inverse DT-CDWT. This algorithm is discussed below.

#### 3.1 Applying Complex Discrete Wavelet Transform

Low frequency content of seismic motions is richer than high frequency one. To get good frequency resolution, we need to decompose earthquake GMs into higher levels. The results of investigated GMs showed that five or six decomposition levels are appropriate for most GMs. In Figure 2, the record of Haramachi station observed during the 2011; Tohoku earthquake<sup>[28]</sup>, is decomposed into five components using DWT. As can be seen, the details 3 & 4 and approximation 4 (low frequency components) are high energy components. This GM has three sharp peaks at its strong motion phase (at  $t=70$  s, 85 s, and 102 s) that should be simulated by heavy-tailed distributions such as the Cauchy distribution. Also, the Gaussian distribution is appropriate for the simulation of its smooth peaks at  $t=36$  s, 54 s, 79 s, and 126 s. So, a robust model for this GM will be a combination of Cauchy and Gaussian distributions, named as the CGB model. The WCs of each level are extracted by applying the DT-CDWT. These coefficients should be simulated by the CGB model.



**Figure 2.** Decomposition of the record of Haramachi station observed during the 2011 Tohoku earthquake<sup>[28]</sup>.

#### 3.2 Modeling the Wavelet Coefficients of Each Level

In the time domain, the amplitude of GMs changes significantly with time. Accordingly, the temporal amplitude of WCs changes similar to that of the target record. There are three distinct phases in the wavelet coefficients of most GMs: 1) in the first phase, the amplitude increases from the zero to a maximum, 2) in stationary phase, the amplitude remains approximately constant, and 3) in the last phase, the amplitude decreases from the maximum to the zero. Such GMs have a rising-descending cycle in their amplitude. Even more complicated GMs have two or more cycles in their amplitude. Most of the stochastic models developed in the previous studies cannot simulate these GMs, because in the previous models, single-component distribution functions are applied to simulate the temporal amplitude of GMs. The GMS of strong earthquakes have also multiple impulsive peaks in the time domain that make it difficult to be simulated by the previous models because in these models the temporal nonstationarity is modeled by short-tailed distributions such as the Gaussian distribution.

In this article, a CGB model is proposed to simulate several ascending-descending cycles and impulsive peaks in WCs. This model simulates sequence ascending-descending cycles and impulsive peaks in the time domain. This model is a weighted combination of multiple heavy-tailed Cauchy distributions and short-tailed Gaussian distributions. It also simulates the steps of the instantaneous cumulative energy curve of GMs. The absolute values of 5th level detail coefficients of the Haramachi GM, along with estimates provided by the CGB model and the Sharbati's model (GB model) are shown in Figure 3. Because this GM has five ascending-descending cycles and three impulsive peaks in its temporal amplitude, a combination of three Cauchy distributions and two Gaussian distributions is used in the CGB model, and a combination of five Gaussian distributions is used in the GB model to simulate the absolute values of WCs. As can be seen in the following figure, the GB model provides an inaccurate estimate of WCs because of the existence of multiple impulsive peaks in the time domain. In contrast, the CGB model simulates multiple impulsive peaks and sudden changes in the absolute of WCs.

The following process generates synthetic WCs and then synthetic GMs with the same characteristics with the target records.

#### 3.3 Extraction of Synthetic Record

After simulating the square of wavelet coefficients

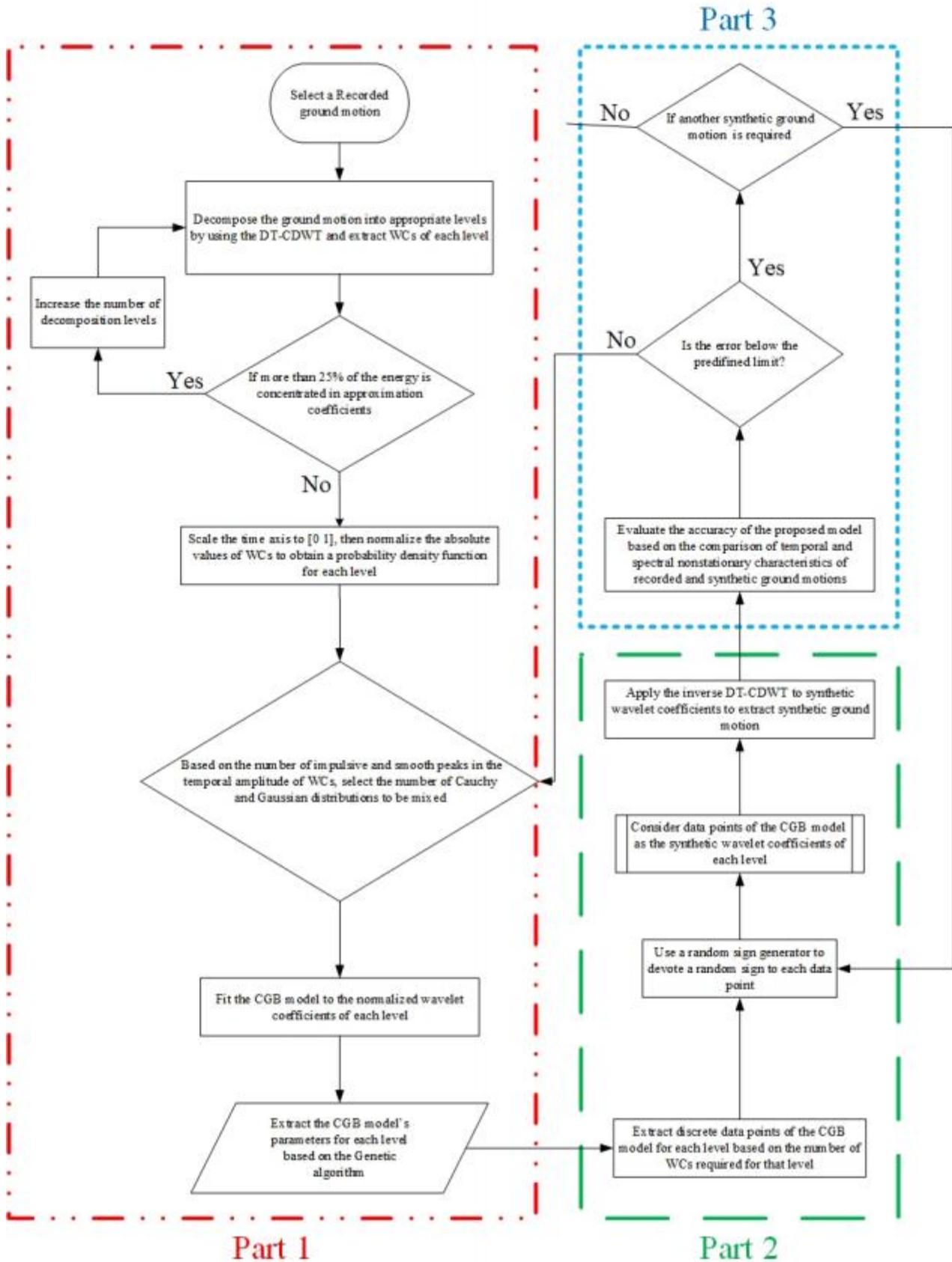
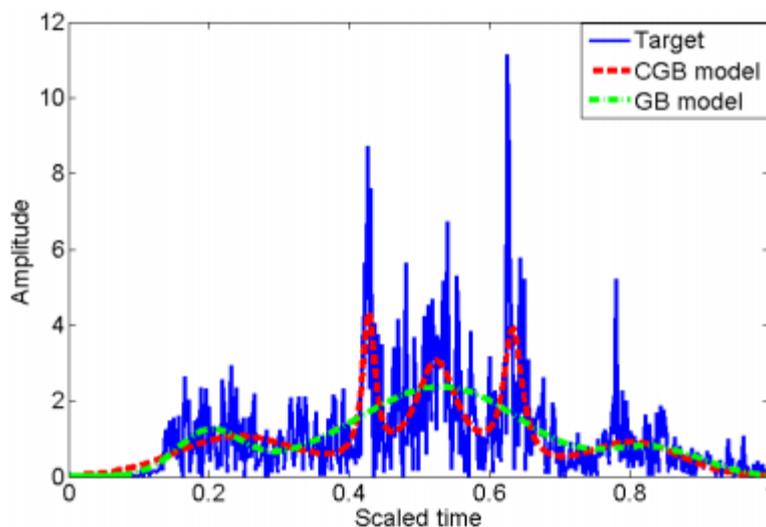


Figure 4. Algorithm of CGB model.



**Figure 3.** Absolute values of 5th level detail coefficients of the Haramachi GM along with the estimates provided by the CGB model and the GB model.

by the CGB model, artificial WCs are generated by the proposed model. The real WCs contain both positive and negative values, but we can only extract positive values by the CGB model, because this model is fitted to absolute values of WCs. Therefore, a random sign generator is used to extract artificial WCs. Therefore, in accordance with real wavelet coefficients, any set of synthetic WCs can be generated by the CGB model. Then, the inverse DT-CDWT is imposed to artificial WCs to extract synthetic GM. So, this model generates several synthetic GMs with the same characteristics as a record. The spectral and temporal nonstationarity of synthetic GMs are similar to those of the record. This conformity will be shown in the time domain, frequency domain, time-frequency domain, and also by response spectrum. The Compact Kernel Distribution (CKD) estimates the energy distribution of GMs. It provides a high resolution image of the energy distribution of GMs, because cross-terms are decreased by a Compact Kernel <sup>[29]</sup>.

The results of CGB model are compared with those of site-based models proposed by Sharbati et al. <sup>[21]</sup> and Vlachos et al. <sup>[19]</sup>. In the former, the Gaussian blend (GB) model is used to simulate WCs. In fact, the GB model is one of the robust stochastic models for the simulation of records. The latter was recently proposed for the simulation of temporal power spectrum of GMs. The algorithm of the proposed model is summarized in Figure 4. There are three sub-blocks in this figure: applying the DT-CDWT to a seismic record and simulation of wavelet coefficients by the CGB model; generation of artificial WCs and then synthetic GM by applying the inverse DT-CDWT; and validation of the proposed method. Two seismic records are

used for the evaluation of the proposed model.

## 4. Simulation Results

We could use several previous models to validate the proposed model, but we want to show important capabilities of the CGB model. So, the results of CGB model are compared with the results of two previous models with more precision and capabilities: Sharbati's model <sup>[21]</sup> and the Vlachos's model <sup>[19]</sup>. For evaluation, two seismic records are used:

A record of the San Ramon Fires station observed during the 1980 Livermore-01 earthquake <sup>[30]</sup>.

A record of the TCU101 station observed during the 1999, Chi-Chi earthquake <sup>[30]</sup>.

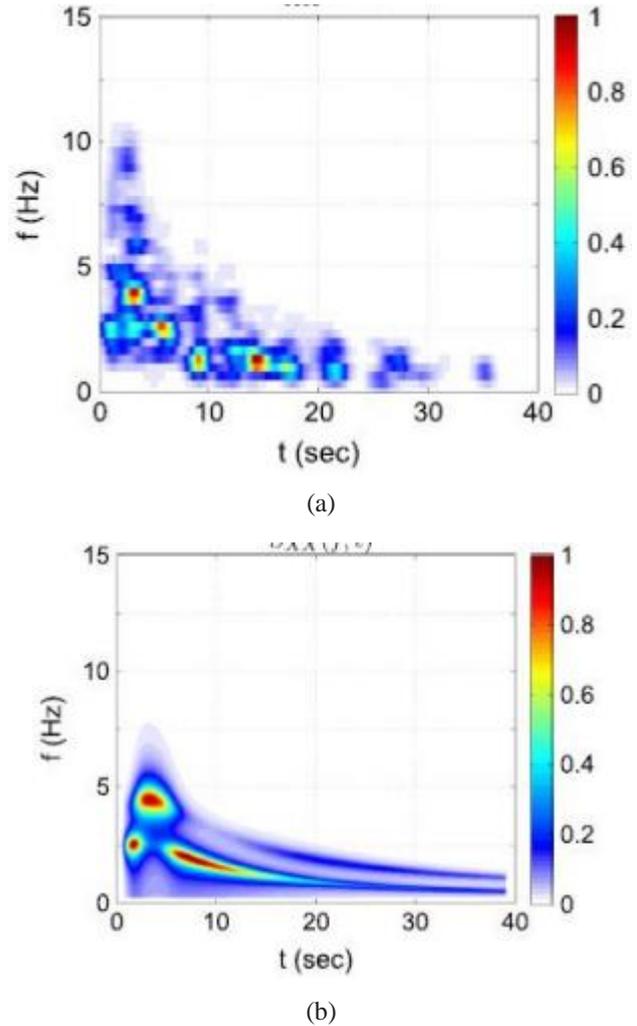
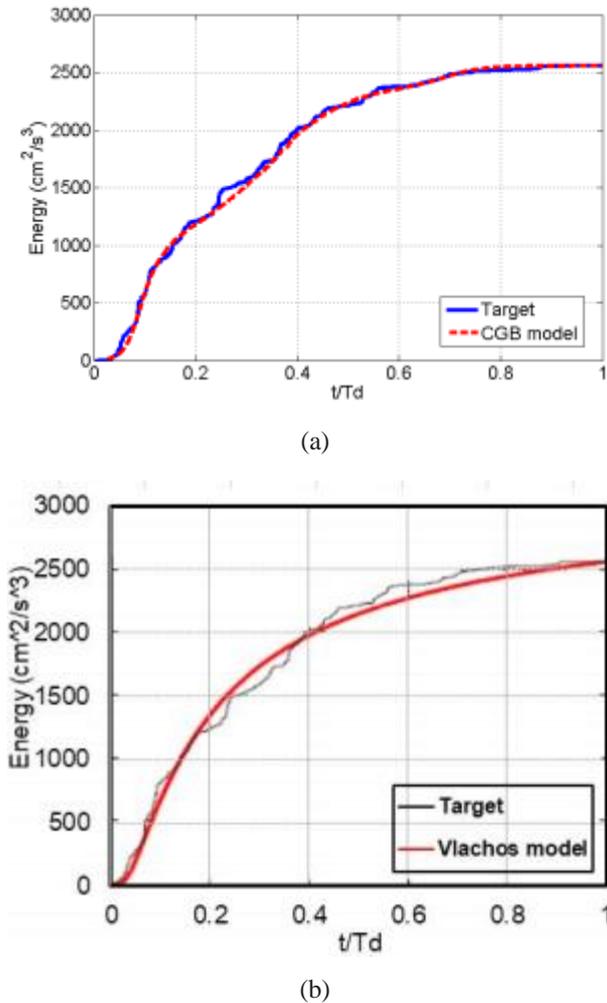
The former is selected to compare the proposed model with the Vlachos model <sup>[19]</sup> and the latter to compare with the Sharbati's model <sup>[21]</sup>. The Livermore GM is classified as a narrowband record and the ChiChi GM as a wideband one. The capability of the CGB model in the estimation of several frequency peaks is evaluated by using the former, and the capability of predicting multiple cycles in the time domain by using the latter.

### 4.1 Comparison with a Previous Model

Here, the results of the CGB model and Vlachos's model <sup>[19]</sup> are compared by using the Livermore GM. In order to compare synthetic and real records, 20 synthetic motions are extracted by these models and the average of their instantaneous cumulative energy curves is compared with the target one in Figure 5. Generation of more synthetic GMs (more than 20 ones) would have no significant effect on the mean instantaneous cumulative energy curve

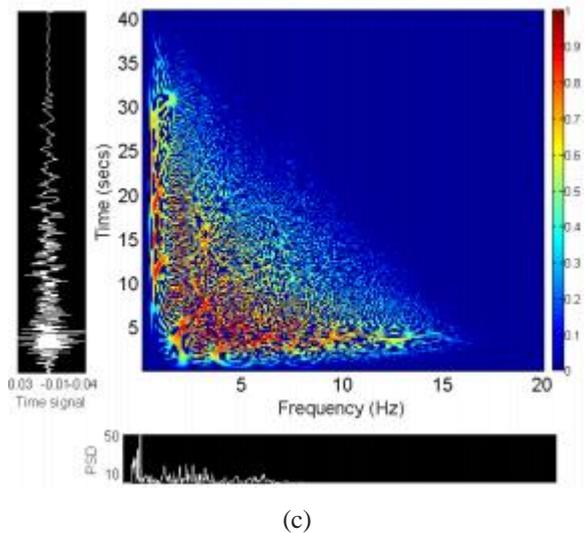
predicted by each model. The time integral of the square of record is defined as the instantaneous cumulative energy. A CGB model that is composed of a Cauchy distribution and a Gaussian distribution is used for the simulation of this GM. According to the results, the CGB model is more accurate than the Vlachos's model in the prediction of this seismic record.

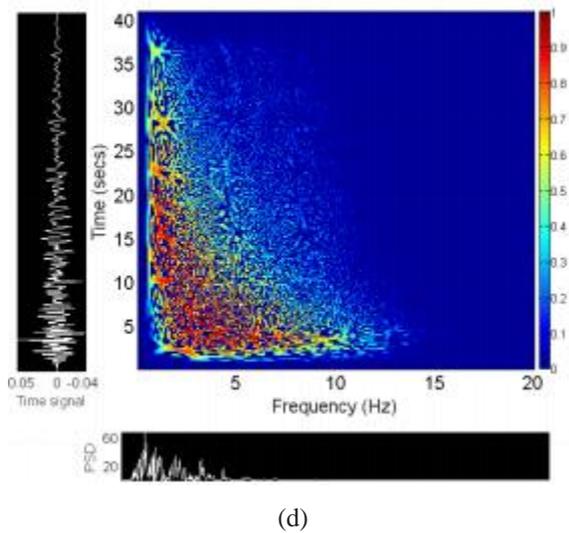
GM. This model can only simulate a record that its energy is concentrated in a short time duration and narrow frequency range. Besides having the capabilities of the Vlachos's model, the CGB model simulates the longtime duration and wide-band GMs.



**Figure 5.** The cumulative energy curve of Livermore GM, along with the results of: (a) the CGB model; (b) the Vlachos's model <sup>[19]</sup>.

Also, the energy distributions predicted by models are shown in Figure 6. Vlachos et al. <sup>[19]</sup> extracted the energy distribution of records by using a method developed by Conte and Peng <sup>[31]</sup>, while here, it is extracted by the CKD. So, the results of two models are compared with the target one separately. By examining Figure 6, it is evident that the CGB model predicts the target distribution more accurately. The energy variations of synthetic motions generated by the CGB model are similar to those of real record. According to the results, the model of Vlachos <sup>[19]</sup> estimates inaccurately the energy distribution of target





**Figure 6.** Comparing the energy distribution of synthetic and real GMs. (a) and (b) are estimated by the method proposed by Conte and Peng<sup>[31]</sup>, (c) and (d) are estimated by the CKD technique<sup>[29]</sup>: (a) the Vlachos's model<sup>[19]</sup>; (b) Target distribution; (c) the CGB model; (d) Target distribution.

## 4.2 Simulation of Impulsive Peaks of Ground Motions

The accuracy of the proposed model (named as the CGB model) and the Sharbati's model<sup>[21]</sup> (named as the GB model) are compared based on the simulation of Chi-Chi record. This record is decomposed into six levels using the DT-CDWT, and then the WCs are simulated by the CGB model and the GB model. For this GM, the energy and frequency band of WCs are given in Table 1. The first components of this record have negligible energy, while most of the energy is devoted to last components. So, decomposition of this record into six levels is the minimum number of required decomposition levels.

Also, a two-component CGB model and a two-component GB model are considered for the simulation of each level (e.g., the combination of a Cauchy distribution and a Gaussian distribution for the CGB model, and two Gaussian distributions for the GB model), because this GM has two ascending-descending cycles or two global peaks in its temporal amplitude. Therefore, the WCs of each level are simulated by five parameters including two location parameters, two variances and a weighting coefficient. The optimal values of model parameters that are extracted by the genetic algorithm are given in Table 2 for the CGB model. The locations of Gaussian and Cauchy distributions ( $\mu$  and  $x_0$ ) are in accordance with the locations of global peaks of the Chi-Chi record.

Figure 7 compares the simulation results of two models

for absolute values of WCs. It is evident that the CGB model estimates global and impulsive peaks of WCs more accurate than the GB model. This superior performance of the CGB model is more pronounced by observing Figure 7(c and d). According to Figure 7(c), the absolute values of 6<sup>th</sup> level detail coefficients have an impulsive peak at  $t/T_d=0.68$ . The GB model could not simulate this impulsive peak, where the CGB model has done it well. Also, the CGB model has provided a more accurate estimate of another global peak at  $t/T_d=0.37$ . Even for the approximation coefficients of level 6 (Figure 7(d)), the CGB model estimates more accurately their impulsive peaks at  $t/T_d=0.67$ , 0.4. It can be concluded that the GB model provides a smooth estimate of WCs by neglecting their impulsive peaks, while the CGB model takes into account sharp and impulsive peaks of WCs.

In Figure 8, the average of cumulative energy curves of synthetic records generated by two models are compared with that of the Chi-Chi GM. This record has a step in its cumulative energy between  $t=34$  s and  $t=56$  s. This step is due to two impulsive peaks at  $t=34$  sec, and  $t=56$  sec. If a model predicts well the temporal amplitude of Chi-Chi GM, this step will be also taken into account by that model. The CGB model has not only simulated the step of Chi-Chi GM, but also provided a more accurate estimate of the target cumulative energy curve at all time points. Also, the results of two models are evaluated based on the root-mean-square error (RMSE). In the estimation of target cumulative energy curve, the CGB model has an RMSE of  $4.8 \times 10^{-10}$ , while it is  $6.9 \times 10^{-10}$  for the GB model. So, the CGB model is 31% more accurate than the GB model.

To evaluate the CGB model further, the elastic and inelastic response spectra estimated by the CGB model and the GB model are compared with the target ones in Figure 9. At all periods, the CGB model estimates target response spectra more accurate than the GB model, where two models have the same number of parameters (e.g., five parameters for each level). Based on the inelastic response spectrum, the CGB model and the GB model have an RMSE of 0.157 and 0.234, and based on the elastic response spectrum, these two models have an RMSE of 0.250 and 0.376 respectively. So, the CGB model is 33% more accurate than the GB model. It should be noted that the GB model underestimates target response spectra at all periods.

As it was noted previously, the energy distribution is the best criterion for evaluating the accuracy of two models. In Figure 10, the energy distribution of synthetic GMs generated by the CGB model and the GB model is compared with that of the Chi-Chi GM. According to Figure 10(a), the Chi-Chi GM has high input energy rates at  $t=28$  s, 55 s. Accordingly, there are two distinct peaks in the en-

**Table 1.** The energy and frequency content of WCs, for the Chi-Chi record.

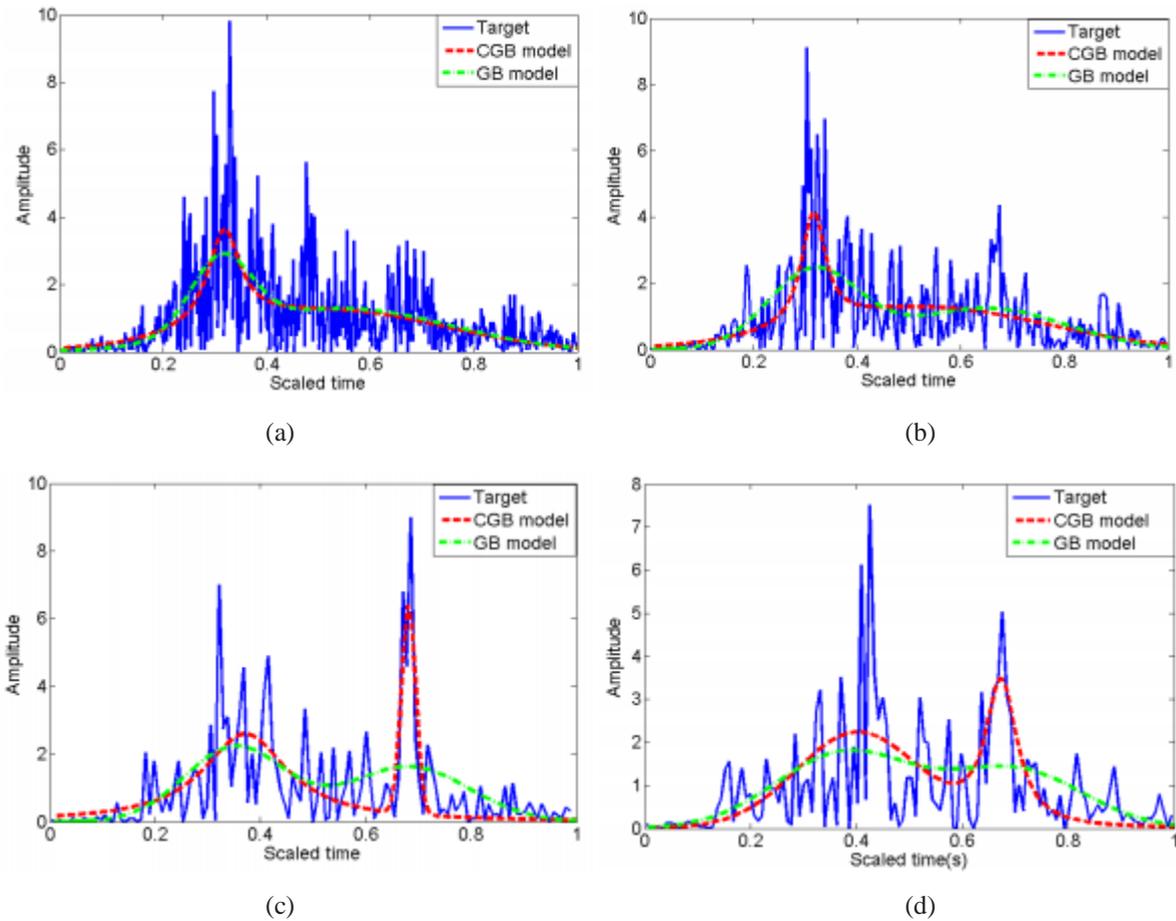
Decomposition level	D - 1	D - 2	D - 3	D - 4	D - 5	D - 6	A - 6
Frequency content (Hz)	25-50	12.5-25	6.25-12.5	3.125-6.25	1.56-3.125	0.78-1.56	0-0.78
Energy of WCs	0.003e+05	0.054e+05	0.511e+05	2.09e+05	4.53e+05	7.11e+05	7.33 e+05

D: decomposition, A: approximation

**Table 2.** Optimal values of the parameters of CGB model, for the simulation of ChiChi record.

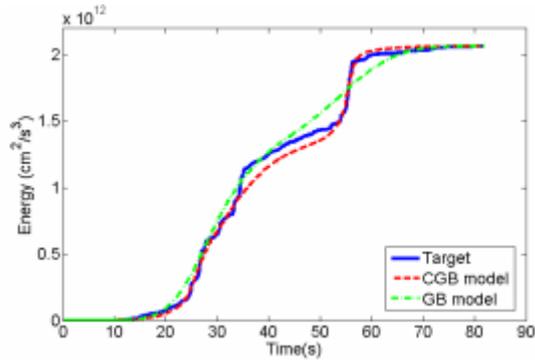
CGB model	Gaussian		Cauchy		Weight
	$\sigma$	$\mu$	$\Gamma$	$x_0$	$\alpha$
D-1	0.0581	0.3097	0.1426	0.6262	0.5146
D-2	0.1063	0.6420	0.0721	0.3092	0.2364
D-3	0.2426	0.4651	0.0177	0.3296	0.8307
D-4	0.2200	0.5268	0.0387	0.3175	0.6502
D-5	0.2266	0.5373	0.0276	0.3154	0.7058
D-6	0.0133	0.6815	0.0972	0.3711	0.2072
A-6	0.1139	0.4007	0.0354	0.6738	0.6237

D: decomposition, A: approximation

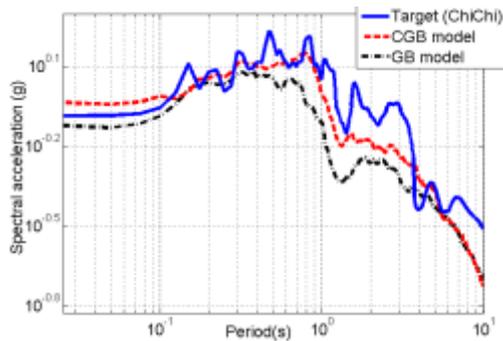


**Figure 7.** Simulation of WCs of the Chi-Chi record by the CGB and GB models: (a) Detail of level 4; (b) Detail of level 5; (c) Detail of level 6; (d) Approximation of level 6.

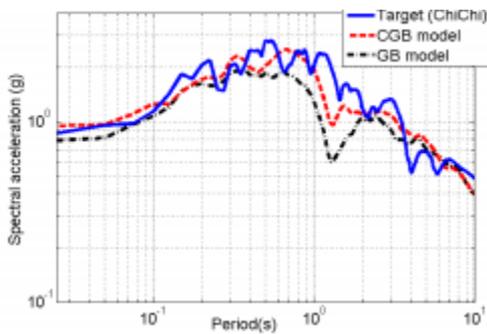
ergy distribution predicted by the CGB model (Figure 10(b)) at these times, while the energy distribution predicted by the GB model (Figure 10(c)) does not have any visible peak. The CGB model provides a more accurate estimate of high energy locations of the target energy distribution (shown by the red color in energy contour plots) by taking into account the sharp and impulsive peaks of real GM. Unlike the energy distribution of real GM, the energy of synthetic ground motion generated by the GB model is distributed in the greater time duration and the wider frequency band.



**Figure 8.** The cumulative energy of Chi-Chi record, along with the results of CGB and GB models.

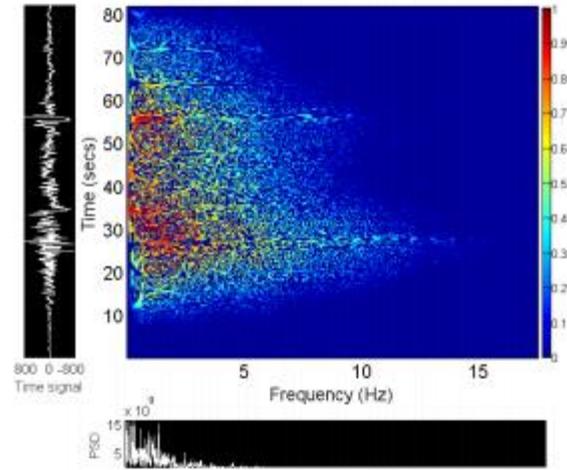


(a)

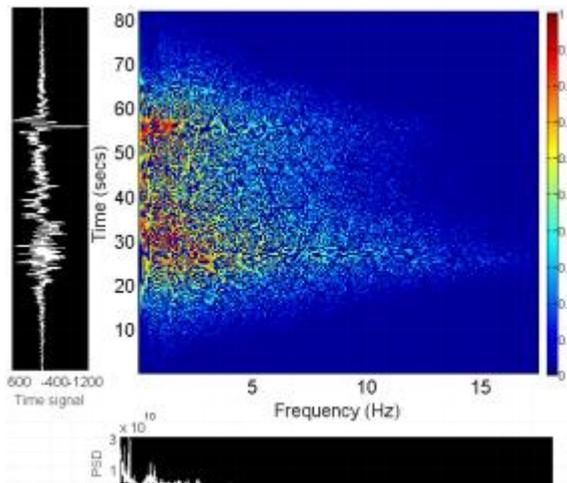


(b)

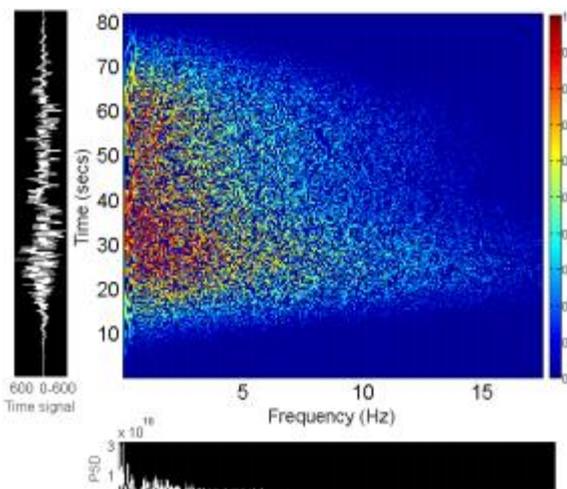
**Figure 9.** Elastic and inelastic response spectra of the Chi-Chi record, along with the results of CGB and GB models: a) Inelastic spectrum for 5% damping ratio and ductility 2; b) Elastic response spectrum.



(a)



(b)



(c)

**Figure 10.** Comparing the energy distributions of synthetic and real records: (a) ChiChi record; (b) The results of CGB model; (c) The results of GB model.

## 5. Conclusions

Based on the Cauchy-Gaussian blend (CGB) model and dual-tree complex discrete wavelet transform (DT-CDWT), a stochastic model is proposed for simulation of wideband and narrowband ground motions (GMs). In this model, the WCs of a record are extracted by DT-CDWT, and then an optimized CGB model is used to simulate them. The accuracy of this model depends on the decomposition levels, the number of Cauchy and Gaussian distributions used in the CGB model, and the method used for the estimation of model parameters. According to the investigated records and previous results, five or six decomposition levels are appropriate. The number of Cauchy and Gaussian distributions depends on the number of global and local peaks in the time domain. Also, the genetic algorithm is used to extract the optimal parameters of the CGB model for each decomposition level.

It was observed that the CGB model simulates well nonstationary characteristics of real records. The results of the CGB model were compared to the results of the Sharbati's model<sup>[21]</sup> and the Vlachos's model<sup>[19]</sup>. These models cannot simulate sharp and impulsive peaks of GMs, while the CGB model overcomes this shortcoming. Two appropriate GMs recorded during past severe earthquakes were selected for the comparison of the models. The results showed that in comparison to previous models, the CGB model provides a more accurate estimate of highly nonstationary GMs, narrowband and wideband records. In this model, the heavy-tailed WCs and impulsive peaks are simulated by the Cauchy distribution, and the smooth and short-tailed ones by the Gaussian distribution. So, the proposed model can well predict concentrated energy distributions and visible peaks in the energy distribution of GMs. Other important capabilities of the proposed model are the simulation of sequence ascending-descending cycles in the time domain, the prediction of several dominant frequency peaks, and the estimation of step and high input energy rate in the cumulative energy of records. These capabilities of the proposed model guarantee the accurate simulation of any type of GM, so this model is suitable for extracting predictive equations.

## Conflict of Interest

There is no conflict of interest.

## References

- [1] Somerville, P.G., Smith, N.F., Graves, R.W., et al., 1997. Modification of empirical strong ground motion attenuation relations to include the amplitude and duration effects of rupture directivity. *Seismological Research Letters*. 68(1), 199-222.
- [2] Luco, N., Bazzurro, P., 2007. Does amplitude scaling of ground motion records result in biased nonlinear structural drift responses? *Earthquake Engineering & Structural Dynamics*. 36(13), 1813-1835.
- [3] Grigoriu, M., 2011. To scale or not to scale seismic ground-acceleration records. *Journal of Engineering Mechanics*. 137(4), 284-293.
- [4] Rezaeian, S., Der Kiureghian, A., 2008. A stochastic ground motion model with separable temporal and spectral nonstationarities. *Earthquake Engineering & Structural Dynamics*. 37(13), 1565-1584.
- [5] Mobarakeh, A.A., Rofooei, F.R., Ahmadi, G., 2002. Simulation of earthquake records using time-varying ARMA (2, 1) model. *Probabilistic Engineering Mechanics*. 17, 15-34.
- [6] Wen, Y.K., Gu, P., 2004. Description and simulation of nonstationary processes based on Hilbert spectra. *Journal of Engineering Mechanics (ASCE)*. 130, 942-951.
- [7] Zhang, Y., Zhao, F., 2010. Artificial ground motion compatible with specified peak ground displacement and target multi-damping response spectra. *Nuclear Engineering and Design*. 240(10), 2571-2578. DOI: <https://doi.org/10.1016/j.nucengdes.2010.05.041>
- [8] Zhang, Y., Zhao, F., Yang, C., 2015. Generation of Nonstationary Ground Motions Compatible with Multi-Damping Response Spectra. *Bulletin of the Seismological Society of America*. 105(1), 341-353. DOI: <https://doi.org/10.1785/0120140038>
- [9] Wang, L., McCullough, M., Kareem, A., 2014. Modeling and Simulation of Nonstationary Processes Utilizing Wavelet and Hilbert Transforms. *Journal of Engineering Mechanics*. 140(2), 345-360. DOI: [https://doi.org/10.1061/\(ASCE\)EM.1943-7889.0000666](https://doi.org/10.1061/(ASCE)EM.1943-7889.0000666)
- [10] Sabetta, F., Pugliese, A., 1996. Estimation of response spectra and simulation of nonstationary earthquake ground motions. *Bulletin of the Seismological Society of America*. 86(2), 337-352.
- [11] Stafford, P.J., Sgobba, S., Marano, G.C., 2009. An energy-based envelope function for the stochastic simulation of earthquake accelerograms. *Soil Dynamics and Earthquake Engineering*. 29(7), 1123-1133. DOI: <https://doi.org/10.1016/j.soildyn.2009.01.003>
- [12] Rezaeian, S., Der Kiureghian, A., 2010. Simulation of synthetic ground motions for specified earthquake and site characteristics. *Earthquake Engineering & Structural Dynamics*. 39(10), 1155-1180.
- [13] Rezaeian, S., Der Kiureghian, A., 2012. Simulation

- of orthogonal horizontal ground motion components for specified earthquake and site characteristics. *Earthquake Engineering & Structural Dynamics*. 41(2), 335-353.
- [14] Medel-Vera, C., Ji, T., 2016. A stochastic ground motion accelerogram model for Northwest Europe. *Soil Dynamics and Earthquake Engineering*. 82, 170-195. DOI: <https://doi.org/10.1016/j.soildyn.2015.12.012>
- [15] Tsioulou, A., Taflanidis, A.A., Galasso, C., 2017. Modification of stochastic ground motion models for matching target intensity measures. *Earthquake Engineering & Structural Dynamics*. pp. 1-22. DOI: <https://doi.org/10.1002/eqe.2933>
- [16] Vetter, C.R., Taflanidis, A.A., Mavroeidis, G.P., 2016. Tuning of stochastic ground motion models for compatibility with ground motion prediction equations. *Earthquake Engineering & Structural Dynamics*. 45, 893-912. DOI: <https://doi.org/10.1002/eqe.2690>
- [17] Yamamoto, Y., Baker, J.W., 2013. Stochastic model for earthquake ground motion using wavelet packets. *Bulletin of the Seismological Society of America*. 103(6), 3044-3056. DOI: <https://doi.org/10.1785/0120120312>
- [18] Huang, D., Wang, G., 2014. Stochastic simulation of regionalized ground motions using wavelet packets and cokriging analysis. *Earthquake Engineering & Structural Dynamics*. DOI: <https://doi.org/10.1002/eqe.2487>
- [19] Vlachos, C., Papakonstantinou, K.G., Deodatis, G., 2016. A multi-modal analytical non-stationary spectral model for characterization and stochastic simulation of earthquake ground motions. *Soil Dynamics and Earthquake Engineering*. 30, 177-191. DOI: <https://doi.org/10.1016/j.soildyn.2015.10.006>
- [20] Vlachos, C., Papakonstantinou, K.G., Deodatis, G., 2018. Predictive model for site specific simulation of ground motions based on earthquake scenarios. *Earthquake Engineering & Structural Dynamics*. 47, 195-218.
- [21] Sharbati, R., Khoshnoudian, F., Ramazi, H.R., et al., 2018. Stochastic modeling and simulation of ground motions using complex discrete wavelet transform and Gaussian mixture model. *Soil Dynamics and Earthquake Engineering*. 114, 267-280. DOI: <https://doi.org/10.1016/j.soildyn.2018.07.003>
- [22] Dak Hazirbaba, Y., Tezcan, J., 2015. Image based modeling and prediction of nonstationary ground motions. *Computers & Structures*. 174, 85-91.
- [23] Tezcan, J., Cheng, J., Cheng, Q., 2014. Modeling and Prediction of Nonstationary Ground Motions as Time-Frequency Images. *IEEE Transactions on Geoscience and Remote Sensing*. (99), 1-8. DOI: <https://doi.org/10.1109/TGRS.2014.2347335>
- [24] Wang, D., Fan, Z., Hao, S., et al., 2018. An evolutionary power spectrum model of fully nonstationary seismic ground motion. *Soil Dynamics and Earthquake Engineering*. 105, 1-10. DOI: <https://doi.org/10.1016/j.soildyn.2017.11.014>
- [25] Kingsbury, N.G., 1999. Image processing with complex wavelets. *Philosophical Transactions of Royal Society A*. 357(1760), 1-16. DOI: <https://doi.org/10.1098/rsta.1999.0447>
- [26] Holland, J.H., 1975. *Adaptation in Natural and Artificial Systems*. University of Michigan Press, Ann Arbor. (2nd Edition, MIT Press, 1992).
- [27] HA, M.P., Kumar, L., Ananthapadmanabha, T., 2014. A novel approach for optimal allocation of a distributed generator in a radial distribution feeder for loss minimization and tail end node voltage improvement during peak load. *International Transaction of Electrical and Computer Engineers System*. 2, 67-72.
- [28] NIED, 2013. Strong-motion seismograph networks (K-NET, Kik-net). National Research Institute for Earth Science and Disaster Prevention. <http://www.kyoshin.bosai.go.jp/> (December 01, 2018).
- [29] Boashash, B., Ali Khan, N., Ben-Jabeur, T., 2015. Time-frequency features for pattern recognition using high-resolution TFDs: A tutorial review. *Digital Signal Processing*. 40, 1-30. DOI: <http://dx.doi.org/10.1016/j.dsp.2014.12.015>
- [30] Pacific Earthquake Engineering Research, 2014. PEER Ground Motion Database - NGA-West2. <http://ngawest2.berkeley.edu/> (December 01, 2018).
- [31] Conte, J.P., Peng, B.F., 1997. Fully nonstationary analytical earthquake ground-motion model. *Journal of Engineering Mechanics (ASCE)*. 12, 15-24.