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Article

An Application for Portfolio Optimization, Risk Sensitivity and Efficient Frontier Visualization in Mathematica

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Abstract: The present study develops a flexible and interactive decision-support application for portfolio optimization, grounded in Modern Portfolio Theory and implemented within the Mathematica computational environment. The tool enables users to construct, analyze, and evaluate investment portfolios dynamically, incorporating real-time sensitivity analysis. In accordance with contemporary portfolio theory, it integrates two principal optimization strategies: (a) the Minimum Variance Portfolio and (b) the Maximum Sharpe Ratio Portfolio. The computational framework ingests real stock market data (Yahoo Finance), from which returns and covariance matrices are calculated. The resulting data serves as inputs for solving the corresponding optimization problems under user-defined constraints. A key feature of the tool is the ability to perform real-time sensitivity analysis with respect to expected returns, as well as to interactively adjust the risk-aversion coefficient, providing users with immediate visual and numerical feedback. Interpretability is enhanced through graphical representations of the Efficient Frontier, overlaid with the optimal portfolios and the Capital Market Line on a unified plot. These visualizations support both educational and practical financial decision-making. Overall, the tool offers a novel contribution by offering a hands-on, visually rich, and analytically rigorous environment for understanding and applying portfolio optimization methods using real-world data.

Keywords: Modern Portfolio Theory; Efficient Frontier; Sharpe Ratio; Risk Aversion; CAPM; Mathematica Software

1. Introduction

The management of investment portfolios remains one of the principal pillars of financial science, with direct implications for both theoretical research and practical application. Since the seminal development of Modern Portfolio Theory by Markowitz (1952) [1], and extending to modern methodologies involving algorithmic optimization, machine learning, and integrated risk analytics, there remains a persistent and evolving need for frameworks that support the rational selection and allocation of investment capital.

In recent years, heightened market volatility—exacerbated by geopolitical instability, macroeconomic shocks, and unpredictable monetary policy shifts—has significantly intensified the importance of robust portfolio optimization techniques. These fluctuations underscore the necessity for dynamic, transparent, and mathematically sound decision-making tools capable of adapting to structural uncertainty and behavioral irregularities in capital markets [2].

The classical Markowitz framework centers on achieving an optimal trade-off between expected return and portfolio risk, with risk typically expressed as the variance of returns. The Efficient Frontier and the principle of diversification constitute foundational tools for decision-making under uncertainty. Moreover, the Sharpe Ratio

(1966) [3] expands this evaluative structure by incorporating excess return relative to the risk-free rate, thereby linking portfolio optimization with market equilibrium theory, as articulated in the Capital Market Line.

The practical and educational need to explore these concepts has led to the development of computational tools that not only provide numerical results but also enable interactive modeling and parametric sensitivity testing. The present study proposes a dedicated application for portfolio optimization in the Wolfram Mathematica programming environment, employing core functions such as Minimize, Manipulate, and ListLinePlot to construct an interactive simulation model. This approach combines theoretical rigor with educational usability, making it effective for both instructional settings and empirical research.

The aim of this article is to present the structure and functionality of this application, analyze the behavior of the two core strategies—the Minimum Variance Portfolio and the Maximum Sharpe Ratio—and assess their robustness under varying assumptions via sensitivity analysis. The study concludes with graphical representations of the Efficient Frontier, the Capital Market Line, and the dynamic evolution of returns, offering a comprehensive visualization of the proposed optimization framework.

Keys contributions of the study:

- Application of portfolio theory in an interactive computational environment: The study implements the theoretical models of Markowitz and Sharpe within the Wolfram Mathematica environment, offering a tool that effectively bridges theory and practice, based on user input (timeframes, parameters) and real-world data from Yahoo Finance.
- Analysis of two core optimization strategies: It presents and compares the Minimum Variance Portfolio and the Maximum Sharpe Ratio strategies, highlighting their practical differences and implications for investment decision-making.
- Integration of sensitivity analysis: The model enables evaluation of the robustness of the optimization results with respect to small variations in expected returns, providing insights into the model's stability.
- Use of parametric control via the Manipulate function: The interactive interface allows real-time exploration of the effects of the risk aversion coefficient (λ) and sensitivity levels, enhancing user understanding of their influence on portfolio allocation.
- Educational and research applicability: The developed framework is suitable for use in both academic teaching and financial research due to its theoretical rigor and computational versatility. It also generates highly informative graphical outputs that visualize return trajectories, the Capital Market Line (CML), and the Efficient Frontier, offering strong interpretative support.
- Demonstration of Mathematica as a powerful tool for financial modeling: The study showcases Wolfram Mathematica as a viable alternative to more widely used platforms (such as Python, R, or Excel) for conducting financial simulations and portfolio optimization.

The structure of the rest of this paper is as follows: Section 2, provides the theoretical and mathematical background of Markowitz's Portfolio Theory and CAPM. Section 3 presents offers a comprehensive look of our framework, showcasing the code, user interaction, and an evaluation of our results. Section 4 discusses potential use cases and propose future work. Section 5 offers a comparison with existing portfolio optimization applications, while Section 6 examines the limitations of the current portfolio management model. Finally, Section 7 concludes the paper by summarizing the key findings. We provide our Mathematica code as Supplementary Materials.

2. Materials and Methods

2.1. Markowitz's Portfolio Theory

Markowitz's Portfolio Theory (Modern Portfolio Theory — MPT) constitutes a groundbreaking framework for constructing investment portfolios, aiming either to maximize expected return for a given level of risk or, alternatively, to minimize risk for a given level of expected return [1]. The core principle of the theory is "efficient diversification", achieved through the strategic selection of assets that are not perfectly correlated with one another.

2.1.1. Historical Framework

Modern Portfolio Theory (MPT) was introduced by Harry Markowitz in 1952 through his seminal article "Portfolio Selection" [4] and further developed with in his 1959 book "Portfolio Selection: Efficient Diversification of Investments" [5]. The discussion of rational behavior under uncertainty in Part IV of the book (1959) begins with a variation of L. J. Savage's axioms [6]. From these axioms, it follows that an investment strategy should be chosen to maximize expected utility over a multi-period investment horizon [6]. In recognition of his contributions to investment theory, Markowitz was awarded the Nobel Prize in Economic Sciences in 1990 [7]. His work laid the foundation for subsequent developments, such as the Capital Asset Pricing Model (CAPM) introduced by Sharpe, Lintner [8], and Mossin [9].

2.1.2. Theoretical Framework

Modern Portfolio Theory (MPT) is built upon the following assumptions: (1) investors behave rationally and make decisions based on logical reasoning; (2) investors are willing to accept higher levels of risk provided that they are adequately compensated with higher expected returns; (3) markets are efficient, meaning that information is fully and promptly disseminated to all market participants; (4) investors can borrow or lend unlimited amounts of capital at a risk-free interest rate; (5) markets are free from transaction costs and taxes; and (6) it is possible to select assets whose returns are independent of other investments within the portfolio.

According to Markowitz, investors should not only consider the expected return of a single asset but also take into account its variance and its correlation with other assets in the portfolio [7,10].

2.1.3. Mathematical Modeling of Portfolio Optimization

Return of Individual Assets

The first step in conducting a Markowitz portfolio analysis is the calculation of the returns of the i individual assets based on their closing prices (with the computation beginning from the second observation day), for each time point t considered in the analysis [11].

The return is calculated using the following formula:

$$r_i = \frac{P_{i,t\text{-}1} - P_{i,t}}{P_{i,t}} \tag{1}$$

where:

- $r_{i,t}$: the return of asset i at time t,
- $P_{i,t}$: the closing price of asset i at time t,
- $P_{i,t\text{-}1}$: the closing price of asset i at time t−1.

Calculation of Average Return

Afterwards, the average return for each asset i is determined by calculating the arithmetic mean of its percentage changes over the observation period [11]. This can be expressed mathematically as follows:

$$E\,(r_i) = \frac{1}{n} \sum_{t=1}^{n} r_{i,t} \tag{2}$$

where:

- $E(r_i)$: the average (expected) return of asset i,
- $\sum_{t=1}^{n} r_{i,t}$: the sum of the returns of asset i over all time periods t,
- n: the number of observations

Construction of the Covariance Matrix

In this stage of the analysis, the covariance between each pair of assets is calculated in order to assess the degree to which their returns co-vary over time. The covariance serves as a critical measure for understanding the interactions among assets within a portfolio and is essential for the construction of the variance-covariance matrix, which underpins the Markowitz portfolio optimization model [11].

The covariance between the returns of assets iii and j is computed using the following formula:

$$\mathrm{Cov}(i,j) = \frac{1}{n-1}\sum_{t=1}^{n}(r_{i,t} - E(r_i))(r_{j,t} - E(r_j)) \tag{3}$$

where:

- Cov(i,j): denotes the covariance between asset iii and asset j.
- $r_{i,t}$, $r_{j,t}$: represent the returns of assets i and j at time t, respectively,
- $E(r_i)$, $E(r_j)$: the average (expected) returns of assets i and j, respectively,
- n: denotes the number of observations.

The full covariance matrix is constructed by systematically calculating all pairwise covariances for the set of assets under consideration. This matrix is symmetric and positive semi-definite and plays a central role in the estimation of portfolio variance and the identification of the efficient frontier.

Expected Portfolio Return and Total Portfolio Risk

For each asset included in the portfolio, a corresponding investment weight is assigned and denoted by w_i. These weights reflect the proportion of the total capital allocated to each asset. The expected return of the overall portfolio is then computed as the weighted sum of the individual expected returns of the constituent assets, according to the following formula [11]:

$$E(r_p) = \sum_{i=1}^{n}(w_i \times E(r_i)) \tag{4}$$

where:

- $E(r_p)$: denotes the expected return of the portfolio,
- w_i: represents the weight of asset i in the portfolio,
- $E(r_i)$: is the expected return of asset
- n: is the number of assets in the portfolio.

Subsequently, the total risk (variance) of the portfolio, denoted as σ_p^2, is estimated using the matrix formulation [11]:

$$\sigma_p^2 = w^T \times \mathrm{Cov} \times w \tag{5}$$

where:

- w^T: is the transposed vector of portfolio weights,
- Cov: is the covariance matrix of asset returns,
- w: is the vector of weights.

2.1.4. Critical Evaluation of Modern Portfolio Theory: Strengths and Weaknesses

Markowitz's portfolio theory provides a comprehensive and mathematically rigorous framework for evaluating the trade-off between risk and return, aiming to achieve diversification within a portfolio. Through effective diversification, it is possible to enhance expected returns without a proportional increase in expected risk [7]. The model is fully compatible with modern computational tools and can be implemented and analyzed using platforms such as the Excel Solver or symbolic computation environments like Wolfram Mathematica, which is utilized in the application presented in this article [12].

However, despite its widespread theoretical acceptance, several key assumptions underlying Markowitz's framework are often violated in real-world conditions, potentially leading to outcomes that deviate from actual market behavior. Specifically:

(a) The assumption of fully rational investors, who are presumed to believe that higher risk always translates into higher expected returns, is not consistently supported by empirical evidence;
(b) The notion of perfect and complete information fails to reflect the presence of information asymmetries that characterize real markets; and
(c) The assumption of zero transaction costs and tax implications overlooks critical factors that can significantly influence optimal portfolio selection [6].

Furthermore, the Markowitz model relies heavily on historical return data to forecast future performance, introducing a level of uncertainty into the optimization process [7]. This reliance may result in unreliable or suboptimal outcomes, particularly during periods of financial crisis or under extreme economic conditions, when past trends fail to predict future dynamics accurately [6].

2.2. Capital Asset Pricing Model — CAPM

The Capital Asset Pricing Model (CAPM) is one of the most fundamental theories in the field of financial economics. The model describes the relationship between risk and the expected return of a security, positing that the expected return of an asset is linearly related to its systematic risk, as captured by the beta coefficient [13].

2.2.1. Historical Framework

Roy (1952) [14] was the first to propose a risk–reward ratio as a means of evaluating the performance of an investment strategy. Building on Roy's ideas, William Sharpe, between 1964 and 1966 [3,15], applied this concept within the mean–variance framework introduced by Markowitz, leading to the development of one of the most widely recognized performance evaluation metrics [16]. Simultaneously and independently, Lintner (1965) [8] and Mossin (1966) [9] expanded upon Markowitz's (1952) [4] foundational work on portfolio theory, jointly contributing to the formulation of the Capital Asset Pricing Model (CAPM).

Their contributions included the introduction of two key assumptions to the original framework:

(a) Homogeneous expectations: Given the asset prices that clear the market at time t = 1, all investors agree on the joint distribution of asset returns from t = 1 to some future time t. Moreover, this distribution is assumed to be the true one—i.e., it is the actual distribution from which observed returns are drawn and against which the model is empirically tested.
(b) Risk-free borrowing and lending: Investors have equal access to borrowing and lending at a risk-free rate, which is constant and independent of the amount borrowed or lent.

William Sharpe, who also introduced the well-known Sharpe Ratio [17], was awarded the Nobel Prize in Economic Sciences in 1990 for his pioneering contributions to asset pricing theory. The CAPM quickly became the dominant tool for estimating the cost of capital and evaluating investment performance [18].

2.2.2. Theoretical Framework

The Capital Asset Pricing Model (CAPM) is founded on the assumption that investors are rational and seek to maximize their utility by selecting portfolios based on expected return and risk. In its standard form, the model assumes that all investors share homogeneous expectations regarding asset returns and have unrestricted access to borrowing and lending at a common risk-free interest rate. As noted by Fama and French (2004) [14], the CAPM relies on the existence of a single, mean–variance efficient "market portfolio" that serves as the benchmark for all investors.

Additionally, the model employs the Sharpe Ratio as a fundamental tool for comparing portfolios based on their return per unit of risk—a concept that is also embedded in the interpretation of the Capital Market Line (CML). The Sharpe Ratio is specifically designed to measure the expected excess return per unit of risk for a zero-investment strategy [17].

The CAPM also draws on Tobin's Separation Theorem, which asserts that all investors, regardless of their individual risk preferences, will choose to invest in a common optimal combination of risky assets—the so-called market portfolio—combined with the risk-free asset. Under these assumptions, differences in expected returns across individual securities are explained solely by their beta coefficients, which quantify the sensitivity of each asset's returns to movements in the overall market, measured as the covariance of the asset with the market portfolio divided by the variance of the market [14].

2.2.3. Mathematical Modeling of Portfolio Optimization

Maximizing the Sharpe Ratio

In addition to portfolio optimization based on a given level of risk aversion, the analysis also considers the case of Sharpe Ratio maximization, i.e., the optimization of the return-to-risk trade-off:

$$\text{Sharpe Ratio} = \frac{E\left(r_p\right) - r_f}{\sigma_p} \tag{6}$$

where:

- $E(r_p)$: the expected return of the portfolio,
- r_f : the risk-free rate,
- σ_p : the standard deviation of the portfolio returns.

The optimization is performed under the following constraints:

- $\sum_{i=1}^{n} w_i = 1$
- $0 \leq w_i \leq 1$

The resulting portfolio is referred to as the market portfolio, as it forms the basis for the Capital Market Line (CML). This portfolio represents the optimal risky portfolio, which, when combined with the risk-free asset, yields the highest possible Sharpe Ratio.

Capital Market Line (CML)

The Capital Market Line (CML) illustrates all feasible combinations of the risk-free asset and the market portfolio. It is graphically represented as a straight line in mean–standard deviation space, and its equation is given by:

$$E(r) = r_f + \frac{E\left(r_m\right) - r_f}{\sigma_m} \times \sigma \tag{7}$$

where:

- $E(r_m)$: the expected return of the market portfolio,
- σ_m : the standard deviation of the market portfolio,
- σ: the total risk (standard deviation) of the combined portfolio.

Equation of CAPM

The corresponding Capital Asset Pricing Model (CAPM) equation is defined as [4]:

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f] \tag{8}$$

where:

- $E(R_i)$: expected return of asset i,
- R_f: risk-free rate,
- $E(R_m)$: expected return of the market portfolio,
- β_i: beta coefficient of asset i, calculated as:

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)} \tag{9}$$

This equation reflects the fundamental CAPM principle that investors are compensated only for systematic risk. Unsystematic (idiosyncratic) risk is assumed to be fully diversifiable and, therefore, not priced in equilibrium [19].

2.2.4. CAPM: Strengths and Weaknesses

The Capital Asset Pricing Model (CAPM) is distinguished by its simplicity and the linear relationship it establishes between risk and expected return, which facilitates the valuation of investment opportunities [16]. The model is widely used in estimating the cost of equity capital and has proven particularly useful in analyses of capital efficiency. Moreover, it offers strong theoretical predictions regarding the quantification of risk and the relationship between expected return and systematic risk [14,20].

However, CAPM does not account for correlations among individual assets and relies on a set of highly restrictive and often unrealistic assumptions. These include the existence of a homogeneous investment horizon, perfect information, and unrestricted borrowing or lending at the risk-free rate—all of which are rarely met in practice [14]. Additionally, the model's foundational assumption of a linear relationship between an asset's beta and its expected return has shown weak empirical support, as several other factors have been found to influence asset returns [14]. Another important limitation lies in the assumption that returns are normally distributed, a condition that does not hold in many practical scenarios, particularly in the context of hedge funds or complex investment strategies. This makes the CAPM potentially unreliable in environments characterized by high volatility or non-Gaussian return distributions.

3. Results

This section presents the results derived from the implementation of the two portfolio optimization models—namely the Minimum Variance model based on Markowitz's theory and the Maximum Sharpe Ratio model—under varying values of key input parameters.

3.1. Minimum Variance Portfolio (Markowitz)

The results obtained from the code execution confirm that the minimum variance portfolio model favors low-risk allocations when the risk-aversion coefficient (λ) takes low values. In such cases, the model prioritizes assets exhibiting low volatility. Conversely, as the investor's aversion to risk increases (i.e., higher λ values), the model tends to include assets with higher expected returns, even at the cost of increased portfolio risk and higher Sharpe Ratio values. The Sharpe Ratio serves here as a performance metric, quantifying the return generated per unit of risk undertaken. These findings reaffirm the theoretical premise that Markowitz's model adjusts the portfolio composition according to the investor's individual risk tolerance.

This dynamic is visually evident in the shape of the Efficient Frontier, which transitions clearly from conservative to more aggressive portfolios as the parameter λ increases.

3.2. Maximum Sharpe Ratio Portfolio

The maximum Sharpe portfolio is identified as the tangency point between the Capital Market Line (CML) and the Efficient Frontier. It represents the optimal trade-off between return and total risk for a rational investor who can borrow or lend at the risk-free rate. As theoretically expected, the optimal weight distribution in this model remains invariant with respect to the investor's risk-aversion coefficient λ, thereby indicating that the Sharpe-optimal portfolio is independent of individual preferences and is determined solely by the characteristics of the investment universe and the selected risk-free rate. Thus, for a given asset universe and timeframe, the Maximum Sharpe portfolio is uniquely defined and identical for all investors.

A typical behavior observed in this model is the concentration of weights on a small subset of available assets. This occurs because the optimization focuses on the individual return-to-risk ratio of each asset rather than solely on their expected returns. As a result, assets with high expected returns but also high volatility may be excluded, while others with modest returns and low variance may receive significant allocation.

Despite its theoretical validity, this concentration introduces practical risks related to sensitivity and lack of diversification. In simple terms, when a portfolio relies heavily on a few assets, a forecasting error in their expected returns or variance can significantly deteriorate overall portfolio performance, increasing its fragility to systemic risk or adverse correlations. This implies that while theoretically optimal, the Sharpe-maximizing strategy may pose challenges regarding portfolio robustness.

3.3. Sensitivity Analysis

The model also investigates the sensitivity of results to perturbations in the expected return vector μ. After applying a ± x% variation to the mean returns, the following observations were made:

(a) The Markowitz minimum variance portfolios displayed relatively stable behavior, with only minor deviations in output performance. This suggests that the model retains its reliability under mild uncertainty, making it useful for investors relying on statistical forecasts or econometric models.
(b) Portfolios derived from the Sharpe optimization approach appeared more sensitive to these return shocks, leading to greater fluctuations in the overall risk-return profile. This sensitivity underscores the higher responsiveness—and potentially higher fragility—of such portfolios in volatile or misspecified environments.

3.4. Graphical Visualization

The implementation includes fully interactive graphical representations of the two optimization models. The user may explore:

(a) Efficient Frontier: A curve plotting optimal risk–return combinations for different values of λ, clearly visualizing the investor's trade-off decisions.
(b) Capital Market Line (CML): A straight line tangent to the Efficient Frontier, originating from the risk-free rate and identifying the Sharpe-optimal portfolio.
(c) Key Portfolios (Markowitz and Max Sharpe): Both portfolios are marked distinctly on the graph using appropriate symbols, tooltips, and legends, enabling intuitive comparison and educational insight.

In addition, the cumulative return plots for each portfolio allow for direct performance comparison based on a hypothetical €1 investment over time. These dynamic visualizations significantly enhance the educational value of the tool, offering a comprehensive decision-support framework that integrates financial theory, numerical results, and immediate interpretability.

3.5. Portfolio Optimization and Risk Analysis in Wolfram Mathematica V(14.1)

The model makes extensive use of core built-in commands such as FinancialData, which enables the automatic retrieval of historical asset prices, and Minimize, which solves the constrained optimization problems central to both the Minimum Variance and Maximum Sharpe Ratio strategies. Interactive parameter exploration is achieved through the use of Manipulate, while ListLinePlot serves for the graphical depiction of cumulative portfolio returns. Data preprocessing and matrix operations are handled using functions such as Transpose, Table, and Mean, supporting the construction of return series and the covariance matrix. This section presents the implementation of a portfolio optimization model in Wolfram Mathematica V(14.1), including a detailed explanation of its structure, user interactions, and computational outcomes based on a hypothetical portfolio. The code is divided into several functional blocks, guiding the user from data input to advanced visualizations and financial interpretations [14,21–23].

To ensure transparency and reproducibility, a detailed description of the user's interaction with the application is provided. The portfolio optimization model implemented in Wolfram Mathematica follows a structured, step-by-step workflow, allowing the user to engage directly with both the data inputs and the optimization outputs. Upon initiating the program, the user is guided through a series of input prompts generated via the Input function. These include: (i) the number of financial assets to be included in the portfolio, (ii) the investment period defined by a start and end date, (iii) the maximum value of the risk aversion coefficient λ for the Minimum Variance model, and (iv) the risk-free rate r_f, which is critical for Sharpe ratio-based optimization.

Following the initial parameter definition, the user is asked to input the ticker symbols of the desired assets. These are stored in a list structure and used to fetch historical price data via FinancialData. Once retrieved, the data are processed into a matrix of daily returns. Statistical properties such as the mean return vector and the covariance matrix, are computed using core Mathematica commands (Mean, Transpose, and Table).

The optimization phase utilizes the Minimize function under standard constraints (i.e., full investment and no short-selling). Two models are computed in parallel: the Minimum Variance Portfolio and the Maximum Sharpe Ratio Portfolio. Through the Manipulate interface, users are given interactive control over the λ parameter and a sensitivity factor, enabling real-time updates of all associated computations and visualizations.

Output metrics include the optimal portfolio weights, expected return, portfolio variance, Sharpe ratio, and cumulative return series. Visual results are presented via ListLinePlot, which illustrates the time evolution of portfolio performance. This user-centric, interactive architecture enhances both the educational value and the practical relevance of the application, bridging theoretical principles with empirical decision-making.

3.5.1. User Interaction and Input Parameters

The first stage of the program involves dynamic interaction with the user, who is prompted to input essential parameters related to the portfolio under investigation (**Figure 1**) [24,25].

```
(*1. USER LOGIN*)
cols = Input["How many assets do you want to import?"];
(*Number of shares to analyze*)startdate = Input["Enter start date (e.g. \"Jan.1,2025\"): "];
(*Data start date*)enddate = Input["Enter end date (e.g. \"Feb.1,2025\"): "];
(*Data end date*)lmax = Input["Maximum risk aversion λ (e.g. 3):"];
(*Upper limit of risk aversion coefficient*)rf = Input["Risk-free rate rf (e.g. 0.02):"];(*Fixed risk-free rate*)

(*IMPORTING TITLES OF ASSETS*)tickers = {};
Do[AppendTo[tickers, Input["Enter ticker for asset " <> ToString[i] <> "(e.g. \"GOOG\"): "]], {i, 1, cols}];
```

Figure 1. Mathematica code that asks for user input (start/end dates, λ, rf).

Specifically, the user must:

(a) Specify the number of assets in the portfolio (≥ 1) (**Figure 2**).

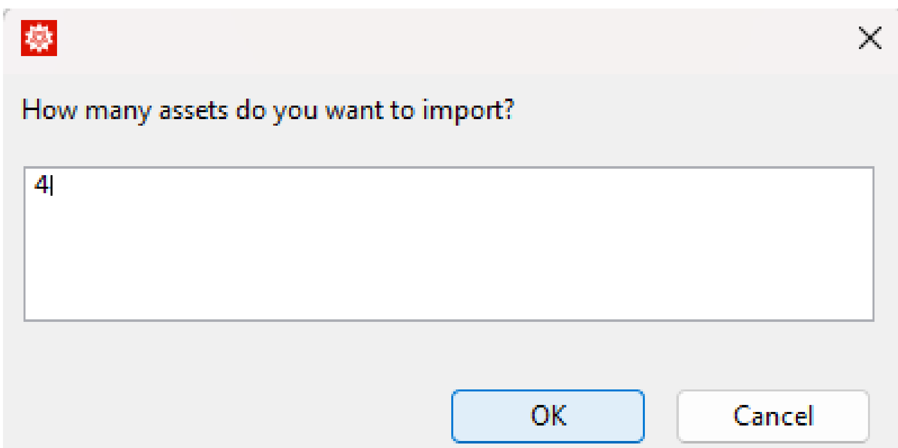

Figure 2. User input window for defining the number of portfolio assets.

(b) Define the timeframe for the analysis by entering the start and end dates as showcased in **Figure 3**.

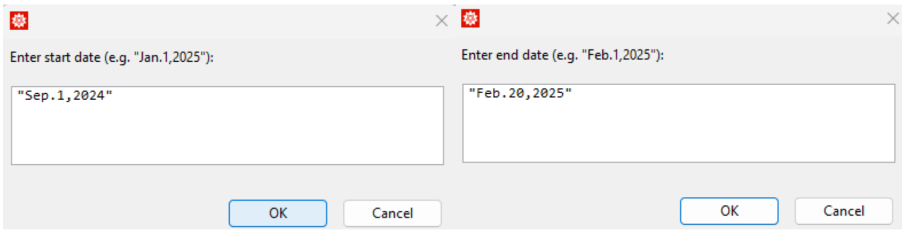

Figure 3. Sequential input windows for defining start and end dates of the investment period.

(c) Set the upper limit for the risk aversion coefficient (λ), reflecting the investor's tolerance to risk (lower values for risk-averse investors and higher for risk-seeking ones) (**Figure 4**).

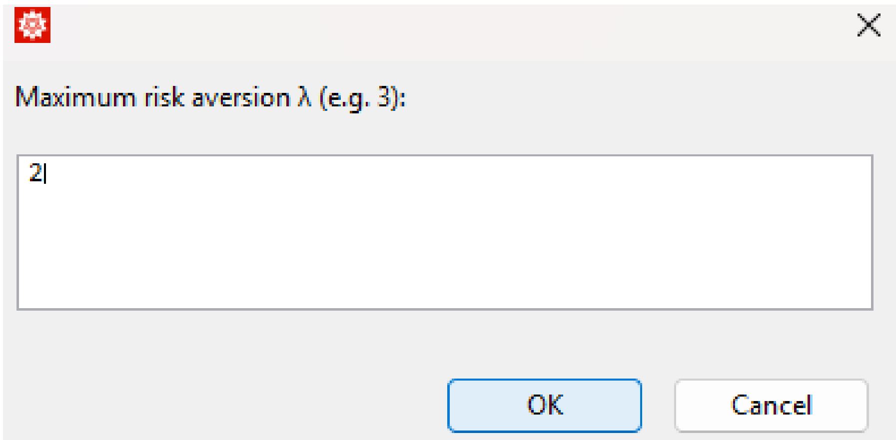

Figure 4. User-defined control for specifying investment risk tolerance.

(d) Provide the risk-free rate, determined by the market in which the investment is made (**Figure 5**).

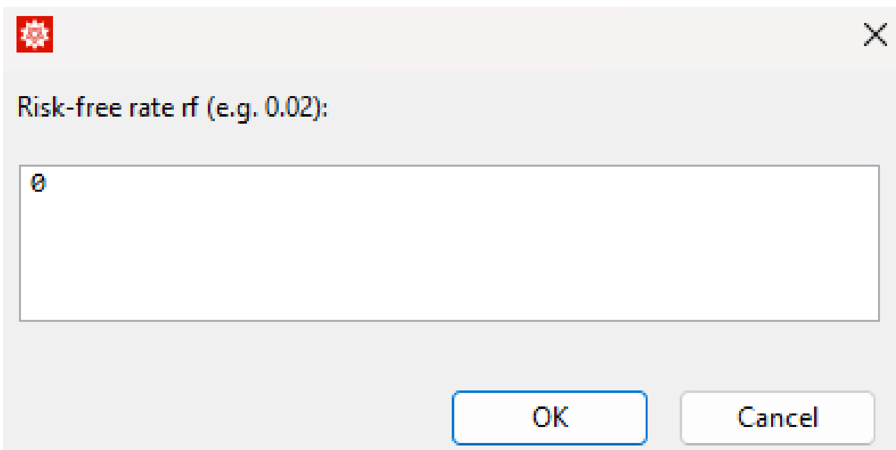

Figure 5. Prompt for setting the risk-free rate used in Sharpe optimization.

(e) Input the asset tickers, as listed on Yahoo Finance, which will be used to retrieve historical data (**Figure 6**).

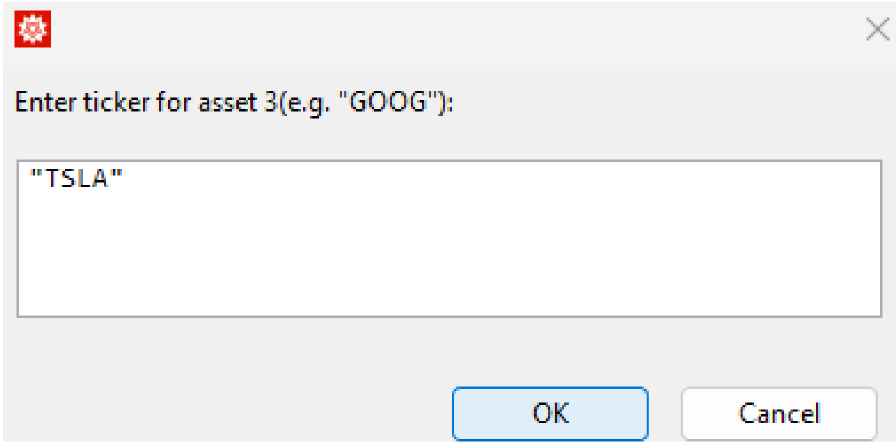

Figure 6. Input window for entering stock ticker symbols as listed on Yahoo Finance.

3.5.2. Data Retrieval

Following the data input, the system proceeds with retrieving historical stock data using Mathematica's built-in FinancialData function, which directly interfaces with Yahoo Finance (**Figure 7**).

```
(*2. FINANCIAL DATA RECOVERY*)
allData = {};
Do[Module[{m, t}, m = FinancialData[ticker, {startdate, enddate}];
  (*Historical price data*)
  t = Which[Head[m] === TemporalData, m["Values"], (*TemporalData object*) Head[m] === TimeSeries, m["Values"],
    (*TimeSeries object*) Head[m] === List && Length[m] > 0 && Head[m[[1]]] === List, Last /@ m, (*List of {date,price} pairs*)
    Head[m] === Missing, Print["Missing data for ", ticker];
    {}, (*Missing data*) Head[m] === $Failed, Print["Failed to get data for ", ticker];
    {}, (*Failed request*) True, Print["Unexpected format for ", ticker, ": ", Head[m]];
    {}   (*Unknown format*)];
  If[Length[t] > 0, AppendTo[allData, t], Print["No data available for ", ticker]];], {ticker, tickers}];
```

Figure 7. Code block executing real-time data acquisition from Yahoo Finance.

3.5.3. Covariance Matrix Construction

Subsequently, the variance-covariance matrix of the portfolio is constructed (**Figure 8**). This matrix includes:

(a) The variances of individual assets along the main diagonal.
(b) The covariances between asset pairs in the off-diagonal elements.

```
(*3. RETURNS & CO-VARIATION CALCULATIONS*)
matrix = Transpose[allData]; (*Price table per day*) rows = Length[matrix];
          |αναστροφή                                  |μήκος

(*Calculating percentage daily returns*)
matrix2 = Table[(matrix[[i + 1, j]] - matrix[[i, j]]) / matrix[[i, j]], {i, 1, rows - 1}, {j, 1, cols}];
           |πίνακας τιμών

mo = Table[Mean[matrix2[[All, j]]], {j, 1, cols}];
     |πίνα··· |μέσος όρος       |όλα
(*Average return of each asset*) cov = Transpose[matrix2].matrix2 / (rows - 1);
                                       |αναστροφή
(*Covariance table*) w = Table[Symbol["w" <> ToString[i]], {i, 1, cols}];
                         |πίνα··· |σύμβολο      |εκτυπωμένη μορφή συμβολοσειράς
(*Weight variables for optimization*)
```

Figure 8. Code segment for constructing the variance–covariance matrix from asset returns.

3.5.4. Portfolio Models and Calculations

The fourth section of the code (**Figure 9**) performs a series of key portfolio optimization calculations, structured into two main models:

(a) Minimum Variance Portfolio (MVP) Model:

- i Total portfolio risk (standard deviation).
- ii Return per unit of risk (Sharpe ratio).
- iii Optimal investment weights for each asset.
- iv Percentage change in average portfolio return for a hypothetical percentage variation (x%) in asset returns.
- v Time series plot of cumulative return over the selected analysis period.

(b) Maximum Sharpe Ratio Portfolio Model:

- i Maximum achievable return.
- ii Corresponding risk level.
- iii Optimal asset allocation for maximizing the Sharpe ratio.

```
(*4. INTERACTIVE OPTIMIZATION & PRESENTATION*)
Manipulate[Module[{e, s2, con, min1, risk1, weights1, sharpe1, returns1, cum1, min2, sharpeVal, weights2, sharpe2, returns2, cum2, avplus,
   avminus, minplus1, minminus1}, (*Constraints:Sum of weights=1 and no negative weights*)con = Join[{Total[w] == 1}, Table[0 ≤ w[[i]] ≤ 1, {i, 1, cols}]];
  e = mo.w; (*Expected portfolio return*)
  s2 = w.cov.w; (*Portfolio variation*)
  (*MODEL 1:MINIMUM VARIATION*)min1 = Minimize[{s2 - l * e, con}, w];
  (*Minimizing objective function*)risk1 = weights1.cov.weights1;
  (*Objective function value*)weights1 = Chop[w /. min1[[2]], 10^-10];
  (*Optimal weights*)sharpe1 = (mo.weights1 - rf) / Sqrt[weights1.cov.weights1];
  (*Sharpe ratio*)returns1 = matrix2.weights1;
  cum1 = FoldList[Times, 1, 1 + returns1];
  (*Cumulative performance per day*) (*SENSITIVITY ANALYSIS (ONLY for MinVariance)*)avplus = (1 + sensit) * mo;
  (*Increase average returns by X%*)avminus = (1 - sensit) * mo;
  (*Decrease average returns by X%*)minplus1 = Minimize[{w.cov.w - l * (avplus.w), con}, w];
  minminus1 = Minimize[{w.cov.w - l * (avminus.w), con}, w];
  (*MODEL 1:MAXIMUM SHARPE RATIO*)min2 = Minimize[{-(e - rf) / Sqrt[s2], con}, w];
  (*Sharpe target reversal for minimization*)sharpeVal = min2[[1]];
  weights2 = Chop[w /. min2[[2]], 10^-10];
  (*Optimal weights for maximum Sharpe*)sharpe2 = (mo.weights2 - rf) / Sqrt[weights2.cov.weights2];
  returns2 = matrix2.weights2;
  cum2 = FoldList[Times, 1, 1 + returns2];
```

Figure 9. Mathematica code segment executing the portfolio optimization models and sensitivity analysis procedures.

3.5.5. Visualization and Interactive Analysis

The fifth section focuses on dynamic visualizations of the above computations (**Figure 10**). Users can interact with sliders and control panels to adjust parameters such as the risk aversion coefficient (λ) and the portfolio's sensitivity, observing in real-time how the asset allocations and key metrics evolve in response to their preferences.

```
(*5. PRESENTATION OF RESULTS*)
 Column[{Style["Model 1: Minimum Variance", Bold, Blue],
 |στήλη   |στυλ                          |έντο··· |μπλε
   Grid[{{"Risk (σ²):", risk1}, {"Sharpe Ratio:", NumberForm[sharpe1, {3, 3}]},
   |πλέγμα                                        |αριθμητική μορφή
     {"Weights:", Grid[Transpose@{tickers, weights1}, Frame → All]}, {"+Sensit [%]:", sensit * 100, "→", minplus1[[1]]},
                  |πλ··· |αναστροφή                |κορνίζα |όλα
     {"-Sensit [%]:", sensit * 100, "→", minminus1[[1]]},
     {"Cumulative Return:", ListLinePlot[cum1, PlotStyle → Red, AxesLabel → {"Day", "Return"}, ImageSize → Medium]}}],
                            |γραμμικό διάγραμμα απ··· |στυλ διαγρά··· |κόκ·· |επικέτες αξόνων    |μέγεθος εικό··· |μεσαίο μέγεθος
   Spacer[15], Style["Model 2: Maximum Sharpe", Bold, Darker@Green],
   |διαχωριστικό |στυλ                         |έντο··· |σκοτειν··· |πράσινο
   Grid[{{"Sharpe Ratio (max):", NumberForm[sharpe2, {3, 3}]}, {"Risk (σ²):", weights2.cov.weights2},
   |πλέγμα                         |αριθμητική μορφή
     {"Weights:", Grid[Transpose@{tickers, weights2}, Frame → All]}}]}]], {{l, 0, "Risk Aversion Coefficient λ"}, 0, lmax, 0.01},
                  |πλ··· |αναστροφή                |κορνίζα |όλα
 {{sensit, 0.05, "Sensitivity ± (%)"}, 0, 0.1, 0.01}]
```

Figure 10. Code section responsible for generating interactive visualizations of portfolio metrics in Mathematica.

A separate section of the code constructs the Capital Market Line (CML) and the Efficient Frontier, providing a visual summary of all efficient portfolios across varying levels of risk aversion (**Figure 11**). On the graph:

i The minimum variance portfolio is marked with a green dot.
ii The tangency portfolio (maximum Sharpe ratio) is marked with a red dot, indicating the optimal intersection of the CML with the Efficient Frontier.
iii A legend is provided showing the corresponding capital weights for each of the two optimal portfolios.

```
(*1. Creating a range of values for the risk aversion coefficient λ*)po = Range[0, lmax, 0.05];

(*Generates λ from 0 to lmax,with a step of 0.05*)

(*2. Setting restrictions on asset weights*)
con = Join[{Total[w] == 1}, Table[0 ≤ w[[i]] ≤ 1, {i, 1, cols}]];
(*Constraints:no short selling and total weight=1*)

(*3. Calculate Efficient Frontier for each λ*)
frontierData = Table[Module[{e, s2, min, opt}, e = mo.w;
    (*Expected portfolio return*)s2 = w.cov.w;
    (*Portfolio variation*)min = Minimize[{s2 - λval * e, con}, w];
    (*Markowitz utility minimization*)opt = w /. min[[2]];
    (*Obtaining optimal weights*){Sqrt[opt.cov.opt], mo.opt}   (*Coordinates {risk,return}*)], {λval, po}];

(*4. Calculating Max Sharpe Portfolio*)
e = mo.w;
s2 = w.cov.w;
minSharpe = Minimize[{- (e - rf) / Sqrt[s2], con}, w];
(*Inversion for minimization*)wSharpe = w /. minSharpe[[2]];
(*Optimal Max Sharpe weights*)pointSharpe = {Sqrt[wSharpe.cov.wSharpe], mo.wSharpe};(*Coordinates {risk,return}*)

(*5. Minimum Variance Portfolio Calculation*)minVariance = Minimize[{w.cov.w, con}, w];
(*Only variation,no expected return*)wMinVar = w /. minVariance[[2]];
pointMinVar = {Sqrt[wMinVar.cov.wMinVar], mo.wMinVar};(*Coordinates {risk,return}*)

(*6. Construct Capital Market Line as a list of points*)cmlLinePoints = {{0, rf}, pointSharpe};
(*From the risk-free rate to the tangency point*)

(*7. Drawing a Graph with a Legend*)
plot = Show[ListLinePlot[{frontierData, (*Efficient Frontier*)cmlLinePoints  (*Capital Market Line*)},
    PlotStyle → {{Purple, Thick}, {Dashed, Blue}}, PlotMarkers → {Automatic, None},
    PlotLegends → Placed[{Style["Efficient Frontier", Purple, Bold], Style["Capital Market Line", Blue, Italic]}, Right],
    PlotLabel → Style["Efficient Frontier with Capital Market Line", 14, Bold],
    AxesLabel → {Style["Risk (σ)", Bold], Style["Return (μ)", Bold]}, ImageSize → Large],
   Graphics[{Red, PointSize[Large], Tooltip[Point[pointSharpe], "Max Sharpe Portfolio"], Darker@Green,
     PointSize[Large], Tooltip[Point[pointMinVar], "Minimum Variance Portfolio"]}]];

(*8. Weighting Tables for the two main portfolios*)
weightsTable =
  Column[{Style["Weights - Minimum Variance Portfolio", Bold, Darker@Green],
    Grid[Transpose@{tickers, Chop[wMinVar, 10^-10]}, Frame → All], Spacer[10], Style["Weights - Max Sharpe Portfolio", Bold, Red],
    Grid[Transpose@{tickers, Chop[wSharpe, 10^-10]}, Frame → All]}];

(*9. Final Presentation (graph and weights together)*)
Column[{plot, Spacer[20], weightsTable}]
```

Figure 11. Code implementation for plotting the Efficient Frontier and Capital Market Line.

3.5.6. Example Application

An illustrative example is provided for a hypothetical portfolio comprising four stocks: Apple (AAPL), Google (GOOG), Tesla (TSLA), and Microsoft (MSFT). The analysis spans the period from 01/09/2024 to 20/02/2025, with a maximum risk aversion coefficient of $\lambda = 2$ and a risk-free rate set to zero.

Figure 12 includes:

➢ The left part of the image, shows the initial portfolio results with risk aversion coefficient λ = 0.
➢ The right part, illustrates how adjustments to the λ, now with λ = 0.5, significantly alter the portfolio composition and capital allocation strategy, yielding us higher returns.

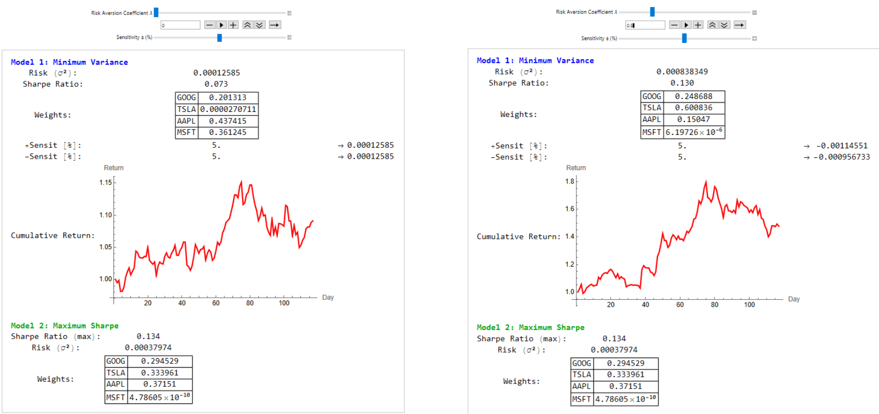

Figure 12. Side-by-side comparison of portfolio composition for λ = 0 and λ = 0.5.

Finally, a comparative graph is presented (**Figure 13**), combining both the Capital Market Line and the Efficient Frontier, highlighting the optimal portfolio configurations under both the Minimum Variance and Maximum Sharpe Ratio frameworks.

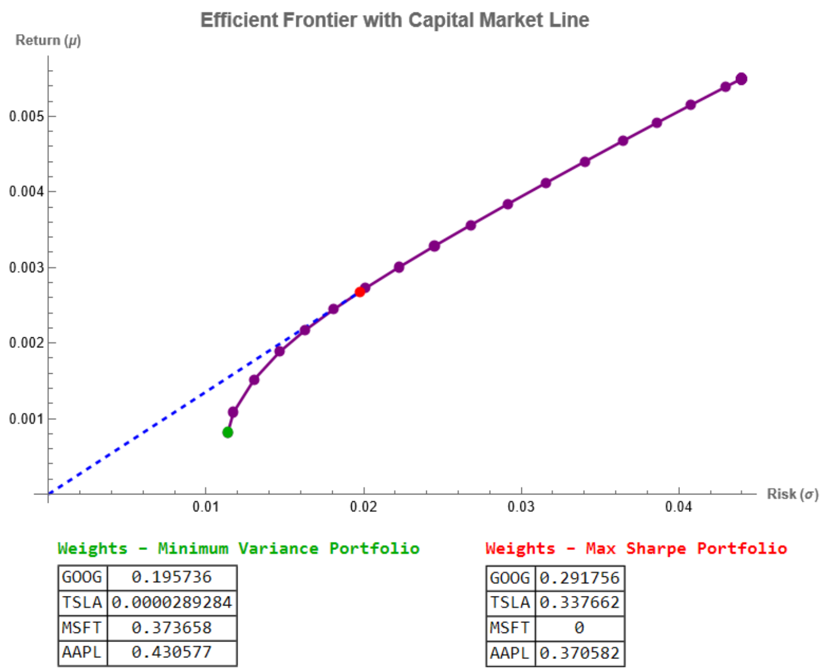

Figure 13. Final summary plot combining Efficient Frontier and Capital Market Line.

The results of the analysis indicate that when the risk aversion coefficient λ ranges between 0 and 2, the variance of the risk minimization model can take on both positive values (e.g., for $\lambda = 0, \sigma^2 = 0.00012585$) and negative values (e.g., for $\lambda = 0.5, \sigma^2 = 0.000838349$). For the same model, as the risk aversion coefficient λ increases, the Sharpe ratio also increases.

Furthermore, it is observed that the model may yield zero weights for one or more of the selected stocks. For instance, when $\lambda = 0$, the weight assigned to Tesla is zero, and when $\lambda = 0.5$, the weight assigned to Microsoft is zero.

Regarding portfolio sensitivity, a decline is observed—sometimes reaching even negative values—with potential asymmetry between the magnitudes of positive and negative sensitivities (e.g., for $\lambda = 0$, sensitivity $= \pm 0.00012585$, and for $\lambda = 0.5$, +sensitivity $= -0.00114551$, and -sensitivity $= -0.000956733$).

Lastly, the Cumulative Return diagram for the examined period shows a sharp increase after approximately the first 40–50 days, followed by gradual stabilization beyond day 80.

In contrast, the Maximum Sharpe model maintains a single strategy (as expected, since it is unaffected by the risk aversion coefficient λ). In the examined case, the model yielded a return of 0.134 with relatively low risk ($\sigma^2 = 0.00037974$). The asset weights for Apple, Google, and Tesla were nearly equal, at approximately 30% of the total investment capital each, while Microsoft stock received a zero weight.

The "Efficient Frontier with Capital Line" diagram identifies the minimum variance portfolio at the point corresponding to $\lambda = 0$, where the Microsoft stock is assigned a weight of zero, while the remaining portfolio stocks range between 29% and 37%. In conclusion, the two curves intersect at a risk level of 0.00037974 (indicated by the red dot on the graph), with the stock weights reflecting those of the Maximum Sharpe model.

3.6. Economic Interpretation and Limitations of MPT Optimization

Assigning a zero weight to specific assets is a frequent outcome in constrained optimization problems, particularly under "long-only" assumptions where short-selling is not allowed. The optimization algorithm prioritizes assets that offer the most favorable risk-return trade-off or provide effective diversification benefits. If an asset has a low expected return or high volatility without contributing to overall portfolio efficiency, it is excluded from the optimal allocation. This exclusion does not imply that the asset is "bad" per se; rather, it means that, under the current input parameters and investor profile, it does not enhance portfolio performance. From a practical perspective, such exclusions may help streamline portfolio construction, reduce transaction costs, and simplify ongoing management.

The risk aversion coefficient (λ) plays a central role in determining portfolio composition. As λ increases, the model increasingly prioritizes risk minimization over return maximization, resulting in allocations dominated by lower-volatility assets. Conversely, a lower λ encourages more aggressive strategies, favoring assets with higher expected returns regardless of their volatility. In effect, varying λ allows the tool to simulate a spectrum of investor profiles, ranging from highly conservative to strongly risk-tolerant. However, in practice, extreme values of λ can produce unstable or unrealistic allocations, underscoring the need for careful calibration. Thus, while λ is valuable for customizing portfolio strategies to different preferences, its use must be balanced to maintain both economic meaning and operational feasibility [26].

The inherent limitations of Modern Portfolio Theory (MPT) and the Capital Asset Pricing Model (CAPM) directly affect the realism and reliability of the tool's outcomes. The assumption of normally distributed returns may lead to risk underestimation, especially during extreme market events. The absence of transaction costs in the model renders the proposed strategies impractical in real markets, where portfolio rebalancing incurs tangible costs. Assuming constant statistical parameters, such as expected returns and covariances, undermines the model's predictive validity and increases the likelihood of overfitting to historical data. Finally, the embedded homogeneity of investor preferences limits the tool's adaptability to diverse investment profiles [27].

4. Discussion

Markowitz's Portfolio Theory remains one of the most fundamental frameworks in modern financial theory. Despite being subject to substantial theoretical and practical criticism, it continues to provide an indispensable structure for understanding the risk–return trade-off and guiding portfolio optimization. Its effective application, however, requires a solid grasp of its underlying assumptions and appropriate adaptation to real-world market conditions.

Similarly, the Capital Asset Pricing Model (CAPM) represents a cornerstone of modern investment theory. While it exhibits both theoretical and empirical limitations, its primary contribution lies in the conceptual foundation it offers for the relationship between risk and expected return, as well as in the development of methodologies for evaluating investment performance. Nevertheless, these limitations underscore the necessity of complementing the CAPM with multifactor models and empirical analysis tailored to actual market data.

The proposed model is well-suited for educational purposes, introductory exploration of efficiency theories, and research-oriented analysis of tailored investment strategies. However, it can also be further extended. Potential future directions include: (a) integration of real-time data to assess strategies under live market conditions, (b) incorporation of alternative risk metrics (e.g., Conditional Value at Risk (CVaR), downside risk, etc.), (c) estimation of parameter forecasting errors (returns, covariances) using Monte Carlo or Bayesian approaches, and the inclusion of machine learning techniques, such as neural networks, to improve portfolio robustness under risk constraints [28], and (d) adaptation to specific portfolio types (e.g., Environmental, Social, and Governance (ESG) [29,30], and cryptocurrencies) through the application of investment screening filters.

Furthermore, the ongoing integration of Artificial Intelligence (AI) and digital technologies into financial services has been reshaping decision-support systems, portfolio automation, and risk forecasting, highlighting the need for adaptable optimization frameworks [31]. One future direction would be the enhancement of our tool via AI-powered return forecasting and anomaly detection, as already emphasized by recent literature on the transformative effects of AI in portfolio management [32].

Moreover, beyond technological advances, structural shifts in capital markets-such as the recent reforms in China's Equities Exchange and Quotation System-have been found to significantly shape innovation strategies and cross-border investment behavior among technology-driven enterprises [33]. These institutional developments are further amplified by the increasingly complex interactions between sustainable finance and digital sectors, as reflected in the dynamic linkages between carbon trading markets and smart technology indices [34]. Accordingly, future extensions of the proposed tool could integrate such multi-layered relationships and environmental-financial interdependencies, enhancing its applicability to resilience-focused and sustainability-oriented investment strategies.

5. Comparison with Existing Portfolio Optimization Applications

The field of portfolio optimization has seen extensive development across both academic and applied settings, resulting in a wide array of tools and platforms. These range from spreadsheet-based solutions to fully programmatic and dynamic systems built in Excel, R, Python, and Mathematica. In this section, we compare the functionality, accessibility, and computational depth of commonly used environments with the framework developed in the current work.

Mathematica offers a unified, mathematically coherent environment for portfolio analysis that significantly surpasses the capabilities of Excel. While Excel is widely used for financial applications due to its interactivity and tabular simplicity, it faces notable limitations in solving complex mathematical optimization problems, such as the Markowitz model, or in conducting sensitivity analysis. In contrast, Mathematica leverages powerful optimization algorithms (e.g., Minimize, symbolic constraints) without requiring external plugins or tools. The Manipulate function enables dynamic parameter exploration in real-time-something that in Excel would require VBA scripting or third-party add-ons. Furthermore, Mathematica automatically handles missing data (Missing[]), while Excel requires manual validation or complex formulas for data cleansing. Statistical visualization (e.g., ListLinePlot) is seamlessly integrated with high mathematical accuracy. While Excel may suffice for basic financial analysis tasks, Mathematica provides a more robust, accurate, and adaptive framework for advanced quantitative finance. Overall, for researchers or analysts needing symbolic computation, interactive modeling, and flexible optimization, Mathematica offers clear advantages over spreadsheet-based approaches.

Mathematica and R are both powerful platforms for financial analysis, yet they differ in design philosophy and in the integration of interactivity and symbolic computation. R provides an extensive ecosystem of statistical tools (e.g., quantmod, PortfolioAnalytics, PerformanceAnalytics) and excels in risk modeling, simulations, and publication-ready graphics via ggplot2. However, for analysts exploring parameter variation-such as changing the risk aversion coefficient (λ)-Mathematica's Manipulate function offers a more intuitive and self-contained GUI, without requiring additional frameworks like R's shiny. Mathematica also provides superior symbolic computation capabilities, making optimization objectives and constraints easier to define and interpret. Although R often outperforms Mathematica in handling large datasets and computation speed (via data.table or parallel processing), Mathematica is ideal for theoretical modeling, symbolic algebra, and interactive simulation [35]. The ability to switch between symbolic and numeric analysis without changing syntax further enhances its value in academic and research settings. Thus, while R is better suited for high-volume data manipulation and statistical depth, Mathematica is preferable for conceptual clarity, visualization, and user-controlled sensitivity experimentation.

Mathematica and Python are both versatile platforms for portfolio optimization and financial modeling. Mathematica integrates mathematical modeling, interactivity (Manipulate), and visualization within a unified environment, eliminating the need for external libraries. On the other hand, Python-using libraries such as pandas, numpy, matplotlib, cvxpy, and scipy.optimize- offers a modular and programmable approach with greater flexibility and control. Python excels in automation, data pipelines, and API integration, making it ideal for production environments. However, Mathematica supports native symbolic optimization, bringing the code closer to formal mathematical notation and making it more intuitive for academic users. Python requires more effort to create interactive applications (e.g., with Dash or Jupyter Widgets), while Mathematica provides built-in visual exploration tools. Although Python's open-source nature and scalability make it appealing for large-scale systems, Mathematica is unmatched in rapid prototyping, theoretical modeling, and clarity of expression. In conclusion, Mathematica is best suited for educational, research, and exploratory scenarios, whereas Python is preferred for industrial applications, data engineering, and large-scale financial infrastructure [36].

In contrast, the portfolio optimization framework developed in this study within the Mathematica environment offers a transparent, modular, and interactive structure. It enables users to control every step of the optimization process – from data import and return calculation to constraint definition and visualization of the Efficient Frontier and Capital Market Line. Moreover, the inclusion of real-time parameter manipulation (e.g., for the risk aversion coefficient or risk-free rate) and sensitivity analysis provides a more dynamic and educationally valuable experience, particularly suited for academic research and teaching.

In summary, while existing tools provide valuable functionality, the proposed Mathematica-based approach strikes a balance between analytical precision, flexibility, and pedagogical clarity, making it especially suitable for environments where explainability and theoretical grounding are essential [24].

6. Limitations of the Current Portfolio Management Model

This paper presents a dynamic portfolio management model implemented within the computational framework of Mathematica V (14.1). However, as with most code implementations, certain limitations exist. In this case, the current version of the model requires that all selected assets originate from the same financial market (e.g., NASDAQ or NYSE) to ensure consistency in data retrieval and comparability of returns. While the code supports ticker symbols containing special characters such as periods (".") or hyphens ("-"), accurate performance depends on unified market conditions and synchronized trading calendars. This constraint restricts the model's applicability across a broader range of financial instruments that utilize non-standard naming conventions in their ticker symbols. Moreover, the increasing presence of AI-driven market behavior introduces a distinction between authentic financial signals and speculative trends-factors not accounted for in classical models [37].

7. Conclusions

The present study developed an interactive portfolio optimization tool within the computational and programming environment of Wolfram Mathematica V (14.1), we provide our code as Supplementary Materials. The tool is based on two portfolio management methodologies: (a) Markowitz's Portfolio Theory, whose primary objective is the minimization of risk, and (b) the Maximum Sharpe Ratio model, which focuses on identifying an efficient portfolio that offers the optimal trade-off between return and risk. Through this application, users can examine investment weight allocations, explore the risk-return relationship, and investigate the behavior of the aforementioned strategies via sensitivity analysis and parameter variation – such as risk aversion (λ or l) and the risk-free rate (Rf).

The findings of the study indicate that the minimum variance portfolio demonstrates greater robustness to changes in input parameters, whereas the maximum Sharpe ratio portfolio, although it maximizes efficiency per unit of risk, exhibits higher sensitivity to fluctuations in expected returns [38]. The graphical representation of the Efficient Frontier, the Capital Market Line, and cumulative daily returns contributed to a deeper and more meaningful theoretical understanding of the risk-return relationship. Furthermore, it highlighted the importance of selecting an investment strategy that aligns with the individual investor's profile.

Supplementary Materials

The following supporting information can be downloaded at: https://mycloud.econ.uth.gr/s/XLHop4TmxM27re6. The Mathematica codes. Mathematica 14.1 or later version is needed.

Author Contributions

Conceptualization, C.M.P.; methodology, E.P.T.; software, C.M.P., E.P.T. and I.P.; validation, C.M.P.; formal analysis, C.M.P., E.P.T. and I.P.; investigation, E.P.T.; data curation, I.P.; writing-original draft preparation, C.M.P., E.P.T. and I.P.; writing-review and editing, C.M.P., E.P.T. and I.P.; visualization, C.M.P.; project administration, I.P. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest

The authors declare no conflict of interest.

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