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Article

Research on Collaborative Governance Strategies for Air Pollution Based on Differential Game Theory

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Abstract: In recent years, air pollution has become an increasingly important issue along with economic and industrial development. To study the interaction between government and enterprises in pollution control, we have developed a differential game model and analyzed their dynamic decision-making process under different governance situations. Three scenarios are considered, including a Nash non-cooperative game, a Stackelberg game led by the government, and a cooperative game with subsidies. The results show that under both the Nash and Stackelberg games, the strategies of government regulation and enterprise emission reduction are completely identical, and the pollution stock remains at a relatively high level. In these two situations, enterprises tend to stay in a passive emission reduction state, so the improvement in pollution governance is limited. However, under the cooperative game, the government encourages enterprises by providing subsidies for emission reduction. As a result, enterprises are more willing to increase their abatement efforts, while the government can reduce part of its regulatory pressure at the same time. Compared with the other two scenarios, the pollution stock decreases more significantly in the cooperative case. These findings indicate that subsidy policies can improve cooperation between government and enterprises and help achieve better environmental governance. And this study may provide some references for the design of air pollution control policies.

Keywords: Pollution Abatement; Game Theory; Differential Game; Environmental Regulation

1. Introduction

Since the advent of the modern era, the widespread development of industrial production has made air pollution a persistent and severe challenge. Beginning with the "Foggy City" of London during the British Industrial Revolution in the 1760s, human society has achieved a leap in productivity while being plunged into an unprecedented air environmental crisis. The large-scale exploitation and combustion of fossil fuels have accelerated the release of carbon sources and sulfides once deeply buried underground, propelling humanity from the "fuelwood age" into the "soot age" and triggering profound ecological tragedies such as the 1952 London Smog Event. With the in-depth advancement of global industrialization and urbanization, air pollution has evolved from the early-stage single-type soot pollution into a complex pollution pattern characterized by the synergy of multiple pollutants—represented by fine particulate matter (PM2.5) and ozone—and cross-regional transmission. Although governments around the world have successively introduced stringent environmental protection legislations and promoted energy transition, under the dual pressures of global climate change and economic development, how to achieve the synergistic emission reduction of air pollutants and greenhouse gases remains a key issue to be addressed in the field of contemporary environmental science and governance. In light of this, an in-depth exploration of the governance paths for air pollution is not only a response to long-standing historical problems but also an inevitable requirement for achieving the goals of sustainable development.

In practical governance, the prevention and control of air pollution cannot be resolved merely by rules and regulations; instead, it is a protracted interest game between regulatory authorities and polluting enterprises. Despite persistent environmental regulations introduced by governments worldwide, air pollution governance still encounters significant practical bottlenecks. Specifically, enterprises are often reluctant to invest in costly emission reduction technologies because such expenditures directly compress their short-term profits. At the same time, regulatory authorities face a continuous rise in enforcement costs, including environmental monitoring, administrative penalties, and technical supervision. Furthermore, the strong regional spillover characteristics of air pollution make it nearly impossible for any single local government to achieve effective governance independently. Balancing economic growth with continuous ecological protection therefore remains a primary dilemma in modern environmental management. The model of "polluting first and governing later" that has prevailed since the Industrial Revolution reveals that enterprises, in pursuing profit maximization, tend to neglect the negative environmental externalities of their activities. In essence, the governance of air pollution is a matter of addressing the negative externalities caused by pollution discharge, and government regulatory strategies directly influence enterprises' motivation for emission reduction. Given that the improvement of air quality is a dynamic process dependent on the accumulation of emission reduction inputs, the decisions made by regulators and polluters often exert intertemporal impacts. Traditional static game models struggle to depict the nonlinear characteristics of pollutant concentration evolution over time and fail to capture the strategic adjustments of all parties along a continuous time axis. However, differential game theory can solve for the "optimal control" and yield specific numerical values for cost inputs. Therefore, applying differential game theory to study the interactive mechanism between government regulation and enterprise pollution discharge enables a more accurate simulation of the dynamic feedback loop among governance inputs, pollutant stock, and environmental benefits, thereby providing a theoretical basis for formulating scientific and sustainable air governance policies. This approach is both theoretically grounded and methodologically applicable. Yi et al. [1] constructed a differential game model for transboundary watershed pollution control and ecological compensation. This model takes the dynamic evolution of cross-regional pollutant transport as the core state variable, emphatically characterizing the game-theoretic interest relationship between upstream and downstream regions in pollution control and ecological compensation. By solving for time-consistent optimal strategies, it achieves a dynamic balance between ecological protection and economic development in the watershed. Yu et al. [2] addressing the cross-regional transport characteristics of transboundary air pollution, proposed a collaborative control strategy model based on differential games. The model introduces the continuous-time concentration of air pollutants as a state variable, accurately depicting the dynamic feedback mechanism between governance investments and environmental quality. This provides a quantitative decision-making framework for collaborative interregional air pollution governance. Wang et al. [3] incorporated public participation into a differential game framework for industrial pollution management and demonstrated that public supervision can effectively improve environmental governance efficiency.

Existing research has extensively applied evolutionary game theory to the field of environmental governance. Zhou et al. [4] constructed a tripartite evolutionary game model involving the government, enterprises, and the public, revealing the core role of reward and punishment mechanisms in pollution control. Wei et al. [5] focused on third-party pollution governance, analyzing the evolutionary trajectory of collaborative strategies between government regulation and enterprises. Sun et al. [6] expanded the evolutionary game analysis framework from the perspective of water environment governance, providing methodological references for water pollution control. Zhao et al. [7] further explored the evolutionary logic of collaborative mechanisms in third-party pollution governance. Song et al. [8] combined complex network theory to analyze the game characteristics of collaborative air pollution governance. Recent studies have also explored collaborative air pollution governance among governments, enterprises, and the public through tripartite evolutionary game frameworks. For example, Wang et al. [9] highlighted the importance of stakeholder coordination mechanisms in improving environmental governance efficiency. However, evolutionary game theory can only describe the trend of group strategy evolution and falls short of capturing the dynamic feedback and intertemporal decision-making in governance behaviors.

In contrast, differential games support the analysis of strategic adjustments in continuous time, enabling the simulation of real-time optimal strategy optimization between governments and enterprises based on current environmental conditions in long-term games, rather than being confined to discrete imitation and trial-and-error processes. Early dynamic game research has laid the theoretical foundation for consistency and optimality in pollution control [10]. Subsequent studies have further extended differential games to intertemporal decision-making and optimal control scenarios: Cai et al. [11] addressed transboundary pollution issues by achieving optimal control solutions for the intertemporal transfer of emission permits through differential games; Xu and Luo [12] further explored optimal environmental policies in dynamic transboundary pollution games, emphasizing the importance of intertemporal coordination mechanisms in pollution governance. These studies collectively validate the core advantages of differential games in characterizing dynamic feedback and intertemporal decision-making in environmental governance.

Differential games have been applied in many different fields, such as water resource price management [13], cooperative interception guidance [14], core technology innovation [15], carbon emission reduction in digital supply chains [16] network security defense [17] and innovative product supply chains [18]. In the context of air pollution control, differential games have also been widely applied: Chen and Li [19] constructed a differential game model for carbon emission reduction that incorporates two types of government contracts—green subsidies and green technologies; Li and Tan [20] compared the effectiveness of cost-sharing and penalty mechanisms in waste incineration pollution control using two differential game models; Luo et al. [21] designed an ecological compensation scheme for air pollution based on differential games; Sun et al. [22] considering government intervention and manufacturer competition, employed differential games to analyze carbon emission reduction effects and pricing strategies. Compared with evolutionary game theory, differential game theory provides a more suitable framework for analyzing air pollution governance characterized by continuous pollutant accumulation and intertemporal decision-making. Evolutionary games primarily focus on the adaptation and evolution of strategies within populations and are effective in explaining long-run behavioral tendencies. However, they are less capable of capturing the dynamic evolution of pollution stock and the optimal control decisions made by individual stakeholders at each point in time. In contrast, differential games explicitly incorporate state variables and continuous-time dynamics, allowing researchers to model the feedback relationship among pollution accumulation, regulatory intervention, and enterprise abatement efforts. This feature is particularly important for air pollution governance because pollutant concentrations evolve continuously and governance decisions often generate delayed environmental effects.

Despite the growing application of differential game theory in environmental governance, several research gaps remain. First, most existing studies focus on a single governance mechanism and seldom provide a systematic comparison among Nash non-cooperative, Stackelberg leader–follower, and cooperative governance structures within a unified analytical framework. Second, the role of government subsidies as a coordination mechanism in cooperative air pollution governance has received limited attention. Third, existing studies rarely investigate how different governance structures influence the long-term evolution of pollution stock, which is a key indicator of environmental quality. To address these gaps, this study develops a unified differential game framework incorporating three governance scenarios and further examines the effectiveness of subsidy policies through sensitivity analysis.

2. Research Framework

The research framework constructed in this paper encompasses three classical game models to comprehensively compare the behavioral characteristics and system performance of regulatory governments and polluting enterprises under different decision-making scenarios. The first is the Nash non-cooperative game scenario [23], which serves as the benchmark decision-making model for this study. It is characterized by the government and enterprises acting as independent, fully rational individuals making simultaneous decisions, with both parties aiming to maximize their own interests without any form of cooperation. In solving this scenario, Hamilton-Jacobi-Bellman (HJB) equations for both the government and the enterprises are constructed separately in continuous time based on dynamic programming principles. First-order partial derivatives are then taken with respect to the control variables of each dynamic system, and the feedback Nash equilibrium strategies for both parties are obtained by simultaneously solving the resulting first-order conditions.

The second is the Stackelberg leader-follower game scenario, which captures decision-making under asymmetric power. The government, leveraging its administrative functions as the leader, formulates incentive and penalty policies first, while enterprises, as followers, react after observing these policies. This scenario is solved using backward induction. Specifically, the enterprise's HJB equation is first established to derive its optimal reaction function, which is then incorporated as a constraint into the government's HJB equation to solve for the final dynamic response.

The third is the cooperative game scenario, which embodies the concept of collective rationality. Here, the government and enterprises break the constraints of individual rationality and engage in joint decision-making with the common goal of maximizing the total system payoff. In solving this scenario, the respective objective payoff functions of the two parties are combined into a single system-wide value function, based on which the system-level HJB equation is constructed. The joint strategies under the Pareto optimal state are then obtained by taking first-order partial derivatives simultaneously with respect to all control variables within the system.

3. Constructions of the Game Models

3.1. Game Subjects and Basic Assumptions

This paper studies the dynamic game behavior between the government and enterprises in the process of air pollution control. The system involves two game subjects: the regulatory government and polluting enterprises. The government imposes constraints on corporate pollution discharge behavior by setting regulatory intensity, while enterprises reduce their pollution discharge levels through emission reduction efforts. Both the government and enterprises are regarded as rational decision-makers, seeking to maximize their respective objective functions under given constraint conditions.

It is assumed that the government aims to maximize social welfare, comprehensively considering the environmental losses caused by pollution and the costs of implementing supervision. Enterprises aim to maximize their own profits, while bearing both the costs of emission reduction and the penalty costs arising from pollution during their production activities. The game between the government and enterprises unfolds over a continuous time interval, where both parties can observe the changes in the system's state variables.

3.2. Basic Assumptions of the Model

Let $x(t)$ denote the stock level of air pollution at time t. The pollution stock evolves dynamically over time, and its change is jointly influenced by factors such as enterprise pollution discharge, emission reduction efforts, government supervision, and natural dissipation.

In addition, stochastic disturbance terms are not incorporated into the pollution stock dynamics. This simplification is adopted because the study focuses on long-term strategic interactions rather than short-term environmental fluctuations. While meteorological conditions, seasonal variations, and monitoring errors may introduce uncertainty into actual pollution dynamics, the deterministic framework allows the core governance mechanisms to be examined more clearly. Future research may extend the model by incorporating stochastic differential game formulations to capture environmental uncertainty.

It is assumed that the emission reduction effort level of the enterprise at time t (i.e., the enterprise's control variable) is $e(t)$, and the regulatory effort level of the government at time t (i.e., the government's control variable) is $g(t)$. The governing equation for the evolution of air pollution is then:

$$\dot{x}(t) = Q - \theta e(t) - \eta g(t) - \delta x(t) \tag{1}$$

Here, Q represents the pollution discharge intensity per unit time in the absence of any governance measures. θ and η denote the efficiency coefficients of the enterprise's emission reduction efforts and the government's regulatory efforts in reducing pollution, respectively. The final term $\delta x(t)$ represents the gradual dissipation of pollutants over time, where δ is the natural dissipation rate. The structure of this model—which concerns pollution stock—is highly representative; similar construction methodologies have been employed in numerous analogous studies.

Based on the above governing equation, we can construct the respective objective functions for the polluting enterprise and the regulatory government. Since both emission reduction efforts and regulatory efforts incur corresponding costs, we assume their costs are quadratic functions of effort: the enterprise's emission reduction cost is $\frac{1}{2}ce^2(t)$, and the government's regulatory cost is $\frac{1}{2}kg^2(t)$, which is similar to Yu et al.'s paper [2]. The quadratic cost structure is widely adopted in environmental economics and differential game studies because it captures the increasing marginal cost associated with higher levels of regulatory and abatement efforts. The parameter c represents the firm's emission reduction cost coefficient, reflecting the economic burden and difficulty of governance that the firm faces during actual pollution control. In real-world environmental governance, firms typically need to invest substantial funds in upgrading cleaner production technologies, purchasing pollution treatment equipment, optimizing energy structures, and innovating green technologies to reduce pollutant emissions. A larger c value means that the firm incurs higher abatement costs for the same level of emission reduction effort. Clearly, this leads to lower emission reduction efficiency and weaker initiative on the part of the firm to voluntarily participate in pollution control. Conversely, a smaller c value indicates that the firm can achieve emission reductions at lower cost, which is more conducive to incentivizing the firm's sustained participation in long-term environmental governance. Moreover, the size of c is likely related to the firm's inherent ability to control pollution. If a firm relies heavily on highly polluting, energy-intensive production methods or lacks advanced environmental technologies, it often needs to invest more in equipment upgrades, pollution treatment, and green technological transformation, resulting in a larger c value. For example, traditional heavily polluted industrial regions such as the U.S. Rust Belt typically have high abatement costs. In contrast, if a firm already has relatively mature cleaner production technologies or engages in high-tech research, it can achieve emission reductions at lower cost, yielding a smaller c value, as seen in Silicon Valley in the San Francisco Bay Area. The parameter k represents the government regulatory cost coefficient, reflecting the economic and administrative costs that the government must bear when implementing environmental regulation. In practice, environmental regulation typically requires investment in environmental monitoring system construction, enforcement inspections, regulatory staffing, and digital governance platforms, among other resources. A larger k value means that the government faces higher regulatory costs to implement environmental regulation, which may weaken its willingness to adopt intensive regulatory strategies. A smaller k value indicates higher government regulatory efficiency and lower regulatory costs, enabling the government to conduct environmental regulation more effectively. The size of k is related to the difficulty the government faces in implementing environmental regulation. If the government's environmental regulatory system is insufficient, monitoring technology is relatively rudimentary, or inspections rely mainly on manual checks, the government will inevitably need to invest more resources, resulting in a larger k value. For example, in a region like West Virginia, with its complex terrain and dispersed pollution sources, reliance on traditional manual field inspections without an automated monitoring network leads to significant resource consumption. Conversely, if the government has established a comprehensive environmental monitoring platform and widely adopts digital regulation, intelligent monitoring, and data sharing technologies, it can achieve efficient regulation at lower cost, resulting in a relatively small k value, as seen in California.

As pollution control improves, the enterprise will generate benefits, which are simplified here as a constant πQ, where π is the enterprise's revenue coefficient. ρ is the discount rate, a positive number equal for both the government and the enterprise. Then, $\phi g(t)x(t)$ represents the regulatory fine imposed by the government on the enterprise based on the pollution stock level—the higher the pollution severity (i.e., $x(t)$) or the greater the government's regulatory intensity (i.e., $g(t)$), the higher the fine, with ϕ being the fine coefficient. Moreover, there is a term $\frac{1}{2}dx^2$, where d is the environmental damage coefficient, reflecting the social welfare loss caused by air pollution. The larger the value of d, the greater the social losses resulting from air pollution; for instance, in densely populated areas or regions with high healthcare costs, d would be substantial. Conversely, the smaller the value of d, the lower the social losses caused by air pollution—for example, in sparsely populated areas, d would likely be relatively small. Thus, the enterprise's objective function is:

$$J_e = \int_0^\infty e^{-\rho t}[\pi Q - \frac{1}{2}ce^2(t) - \phi g(t)x(t)]dt \tag{2}$$

Similarly, the government's objective function is:

$$J_g = \int_0^\infty e^{-\rho t}[-\frac{1}{2}dx^2(t) - \frac{1}{2}kg^2(t)]dt \tag{3}$$

Here, the government has no explicit revenue because this study focuses primarily on the regulatory and incentive functions of environmental penalties rather than their fiscal contribution. Although environmental fines may generate revenue for governments in some regulatory systems, such revenues are generally intended to deter non-compliant behavior rather than serve as a major source of public finance. Compared with broader government revenues such as taxation and fiscal transfers, environmental penalty revenues are assumed to be relatively small and therefore can be neglected in the present model. This simplification allows the analysis to concentrate on the strategic interaction between governments and enterprises while maintaining analytical tractability. Future studies may explicitly incorporate penalty revenues into the government's objective function to improve empirical realism.

4. Solution of the Models

4.1. Nash Non-Cooperative Game

In the context of a non-cooperative game, the polluting enterprise and the regulatory government simultaneously choose their respective control variables, $g(t)$ and $e(t)$, in order to maximize their individual discounted payoff functions. Neither party considers the reaction behavior of the other. The solution of the model yields the static equilibrium solution for the government and the enterprise under the Nash non-cooperative game.

First, it is necessary to construct the Hamilton-Jacobi-Bellman (HJB) equations for both players based on the aforementioned objective functions. In these equations, the time variable t is omitted from the functions. According to the relevant theory, the HJB equation for the government is:

$$\rho T_g(x) = \max_{g(\cdot)}\{-\frac{1}{2}dx^2 - \frac{1}{2}kg^2 + \frac{\partial T_g(x)}{\partial x}[Q - \theta e - \eta g - \delta x]\} \tag{4}$$

The HJB equation for the enterprise is:

$$\rho T_e(x) = \max_{e(\cdot)}\{\pi Q - \frac{1}{2}ce^2 - \phi gx + \frac{\partial T_e(x)}{\partial x}[Q - \theta e - \eta g - \delta x]\} \tag{5}$$

Both equations contain a derivative term, which represents the derivative of each party's payoff under optimal governance. Here, T denotes the maximum discounted payoff that either party can obtain given the system state x. For any $Q > 0$, the equations are continuously bounded and differentiable.

Next, we need to solve Equations (4) and (5). In the non-cooperative game, the enterprise and the government must each maximize their own optimal payoff functions. Let the HJB equation for the government be H_g, and the HJB equation for the enterprise be H_e. First, it is necessary to write the adjoint equations and the necessary conditions for the state variable $x(t)$ and the control variables $g(t)$ and $e(t)$, as follows:

$$\frac{\partial H_g}{\partial x} = -dx - \frac{\partial T_g(x)}{\partial x}\delta \tag{6}$$

$$\frac{\partial H_g}{\partial g} = -kg - \frac{\partial T_g(x)}{\partial x}\eta \tag{7}$$

$$\frac{\partial H_e}{\partial x} = -\phi g - \frac{\partial T_e(x)}{\partial x}\delta \tag{8}$$

$$\frac{\partial H_e}{\partial e} = -ce - \frac{\partial T_e(x)}{\partial x}\theta \tag{9}$$

Equations (7) and (9) are the necessary conditions. Setting them equal to zero, we can solve for:

$$g = -\frac{\frac{\partial T_g(x)}{\partial x}\eta}{k} \tag{10}$$

$$e = -\frac{\frac{\partial T_e(x)}{\partial x}\theta}{c} \tag{11}$$

There are still unknown parameters in g and e. Therefore, to truly solve for g and e, the derivatives within them must be derived. Consequently, from Equations (6) and (8), we obtain the following. Here, the equivalent transformation of the HJB equation is applied:

$$\frac{\partial H_g}{\partial x} = \rho\frac{\partial T_g(x)}{\partial x} = -dx - \frac{\partial T_g(x)}{\partial x}\delta \tag{12}$$

$$\frac{\partial H_e}{\partial x} = \rho\frac{\partial T_e(x)}{\partial x} = -\phi g - \frac{\partial T_e(x)}{\partial x}\delta \tag{13}$$

From Equations (12) and (13), we can solve for:

$$\frac{\partial T_g(x)}{\partial x} = -\frac{dx}{\rho+\delta} \tag{14}$$

$$\frac{\partial T_e(x)}{\partial x} = -\frac{\phi g}{\rho+\delta} \tag{15}$$

Substituting these two derivative values into Equations (11) and (12) just solved, we can obtain:

$$g* = \frac{dx\eta}{(\rho+\delta)k} \text{ and } e* = \frac{\phi g\theta}{(\rho+\delta)c}$$

Here, $g*$ and $e*$ represent the optimal strategies for the government and the enterprise. Since the pollution stock level x is the core indicator in this study, the optimal strategies should be substituted back into Equation (1). By setting $\dot{x} = 0$, making the pollution stock level no longer change over time, the final result is as follows:

$$x* = \frac{Q - \frac{\phi g\theta^2}{(\rho+\delta)c} - \frac{dx\eta^2}{(\rho+\delta)k}}{\delta}$$

4.2. Stackelberg Game

In the actual process of air pollution control, the government usually holds a dominant position in policy formulation and regulation, while enterprises choose their own emission reduction behaviors under the given intensity of government regulation. Therefore, this paper further constructs a Stackelberg leader-follower game model between the government and the enterprise to better reflect the real-world situation, in which the government acts as the leader and the enterprise acts as the follower.

Under this game structure, the government first sets the governance intensity $g(t)$. After observing the government's strategy, the enterprise chooses its optimal emission reduction effort $e(t)$. Both parties make dynamic decisions based on the pollution stock $x(t)$, forming a Markovian Stackelberg game. The dynamic evolution process of the pollutant is still expressed by Equation (1), where the meaning of each parameter remains consistent with that in the Nash non-cooperative game.

We first analyze the follower of the game—the enterprise. Given the government's governance strategy $g(t)$, the enterprise aims to maximize its discounted payoff J_e. The HJB equation for the enterprise is then:

$$\rho T_e = \max_e\{\pi Q - \frac{1}{2}ce^2 - \phi gx + T_e{}'(x)[Q - \theta e - \eta g - \delta x]\} \tag{16}$$

Similarly, let the HJB equation for the enterprise be H_e. Then, derive the necessary condition equation with respect to the enterprise's control variable e:

$$\frac{\partial H_g}{\partial e} = -ce - T_e{}'(x)\theta \tag{17}$$

Then, the optimal emission reduction strategy for the enterprise can be solved as:

$$e*(t) = -\frac{T_e{}'(x)\theta}{c} \tag{18}$$

Next, the government anticipates that the enterprise will adopt $e*(t)$ as its strategy. Its HJB equation is then:

$$\rho T_g = \max_g\{-\frac{1}{2}dx^2 - \frac{1}{2}kg^2 + T_g{}'(x)[Q + \frac{\theta^2}{c}T_e{}'(x) - \eta g - \delta x]\} \tag{19}$$

In the above equation, the value of e is substituted with the enterprise's optimal emission reduction strategy derived from Equation (18). Next, let the government's HJB equation be H_g, and derive the necessary condition equation with respect to the government's control variable g:

$$\frac{\partial H_g}{\partial g} = -kg - T_g{}'(x)\eta \tag{20}$$

The optimal regulatory strategy for the government is solved as:

$$g*(t) = -\frac{T_g{}'(x)\eta}{k} \tag{21}$$

Additionally, we need to write the adjoint equations for the two HJB equations, as follows:

$$\frac{\partial H_g}{\partial x} = -dx - T_g{}'(x)\delta \tag{22}$$

$$\frac{\partial H_e}{\partial x} = -\phi g - T_g{}'(x)\delta \tag{23}$$

At this point, we need to apply the equivalent transformation of the HJB equation once again, in a form that is completely consistent with the previous Equations (12) and (13). Consequently, we can obtain the optimal strategy choices for the government and the enterprise:

$$g** = \frac{dx\eta}{(\rho+\delta)k}, e** = \frac{\phi g\theta}{(\rho+\delta)c}$$

Upon comparison, it is found that the strategies obtained for both parties under this scenario are the same as those obtained under the Nash non-cooperative game. Therefore, in this case, the pollution stock should also be identical to that in the previous scenario, which is:

$$x** = \frac{Q - \frac{\phi g\theta^2}{(\rho+\delta)c} - \frac{dx\eta^2}{(\rho+\delta)k}}{\delta}$$

4.3. Cooperative Game

In the aforementioned non-cooperative game and Stackelberg game, both the government and the enterprise make decisions with the goal of maximizing their own payoffs. However, in the actual process of pollution control, the government and the enterprise can often achieve improved overall governance efficiency through a cooperative mechanism, which is often the best system. For example, Ding et al. [24] constructed a differential game model for ecological compensation based on cross-regional government-enterprise cooperation and demonstrated that cooperative mechanism can maximize the overall system benefits in pollution control. Therefore, this paper further constructs a cooperative game model between the government and the enterprise, assuming that both parties form a unified decision-making objective through negotiation to maximize the total payoff of the system.

In this case, some changes need to be made to the model. The government will provide subsidies to the enterprise for its genuine emission reduction efforts. Here, we denote the subsidy item as $s(t)$, which represents the government's unit subsidy amount per unit of time for the enterprise's emission reduction efforts. The actual subsidy amount is also related to the enterprise's emission reduction effort level. Therefore, the subsidy amount is set as $s(t) = \alpha e^2(t)$, where α is the subsidy coefficient. Consequently, the objective function under the cooperative game is:

$$J_c = \int_0^\infty e^{-\rho t}[\pi Q - \frac{1}{2}dx^2(t)\frac{1}{2}kg^2(t) - \frac{1}{2}ce^2(t) + \alpha e^2(t) - \phi g(t)x(t)]dt \tag{24}$$

Then, the HJB equation for the cooperative game can be constructed as:

$$\rho T = max\{\pi Q - \frac{1}{2}dx^2 - \frac{1}{2}kg^2 - \frac{1}{2}ce^2 + \alpha e^2 - \phi gx + T'(x)[Q - \theta e - \eta g - \delta x]\} \tag{25}$$

At this point, there are three control variables, namely $s(t)$, $g(t)$, and $e(t)$. Let H denote the HJB equation, and derive the necessary conditions for them, as follows:

$$\frac{\partial H}{\partial g} = -kg - \phi x - T'(x)\eta \tag{26}$$

$$\frac{\partial H}{\partial e} = -ce + 2\alpha e - T'(x)\theta \tag{27}$$

The solution is:

$$g***=-\frac{\phi x+T'(x)\eta}{k},e***=-\frac{T'(x)\theta}{c-2\alpha}$$

Finally, write the adjoint equation of the cooperative game HJB equation:

$$\frac{\partial H}{\partial x}=-dx-\phi g-T'(x)\delta \tag{28}$$

Here, the equivalent transformation of the HJB equation is used once again to derive the expression for $T'(x)$ and substitute it back into the optimal strategies of both parties, yielding the optimal strategies as follows:

$$g***=-\frac{\phi x}{k}+\frac{(dx+\phi g)}{k(\rho+\delta)},e***=\frac{(dx+\phi g)\theta}{(\rho+\delta)(c-2\alpha)}$$

In this case, since both parties aim to maximize the overall system benefit, it is evident that the pollution stock in this type of game is lower than that in the previous two scenarios.

5. Results and Discussion

To further validate the model derivation results and compare the dynamic performance of air pollution control under different game mechanisms, this paper conducts numerical simulation analysis based on MATLAB (Matrix Laboratory) for three scenarios: Nash non-cooperative game, Stackelberg leader-follower game, and cooperative game.

Based on the practical context of air pollution control and the stability requirements of the model, the relevant parameters are set as follows. Some parameters in this paper are selected with reference to related literature [25]. However, we will inevitably need to make some changes to the parameter assignments since we cannot use them exactly as they are. "The baseline pollution emission intensity is set to $Q=8$. The corporate emission reduction efficiency coefficient and the government regulatory efficiency coefficient are set to $\theta=0.4$ and $\eta=0.3$, respectively. The natural pollutant decay rate is set to $\delta=0.15$. Meanwhile, the corporate emission reduction cost coefficient and the government regulatory cost coefficient are set to $c=4$ and $k=6$, respectively. The government penalty coefficient is set to $\phi=0.5$, and the government subsidy coefficient under the cooperative governance scenario is set to $\alpha=0.6$. In addition, the environmental damage coefficient is set to $d=15$, and the discount rate is set to $\rho=0.05$. The initial pollution stock is set to $x(0)=100$, and the simulation time horizon is set to $T=60$. The above parameters not only satisfy the system stability conditions but also ensure that the pollution stock remains non-negative throughout the simulation, which is consistent with real-world environmental governance principles and economic meaning.

In accordance with the Equation (1), the dynamic evolution processes of pollution stock under the three game scenarios are simulated respectively, and the results are shown in **Figure 1**.

Figure 1 shows that under all three game mechanisms, the pollution stock gradually decreases over time and eventually stabilizes. The trajectories of pollution stock under the Nash non-cooperative game and the Stackelberg leader-follower game are both stabilizing at relatively high levels, indicating that under non-cooperative conditions, the government's first-mover advantage is insufficient to significantly improve governance efficiency due to limited corporate emission reduction incentives. In contrast, under the cooperative game scenario, the pollution stock declines more rapidly and stabilizes at a much lower level, demonstrating that government subsidies effectively incentivize firms to increase emission reduction efforts, shifting them from passive compliance to active participation. Therefore, the cooperative governance mechanism can reduce pollution stock more effectively than non-cooperative models.

An interesting result of this study is that the Nash and Stackelberg games generate identical equilibrium strategies and pollution trajectories. This outcome stems from the structure of the model. Although the government acts as the leader in the Stackelberg game, the enterprise's response is determined by a linear penalty mechanism and quadratic abatement cost function. Consequently, the government's first-mover advantage does not create additional incentives for firms to increase emission reduction efforts beyond the Nash equilibrium level. In other words, administrative authority alone is insufficient to improve environmental outcomes when enterprises continue to bear the full cost of abatement. This finding highlights the importance of economic incentives, such as subsidies, in stimulating proactive environmental behavior.

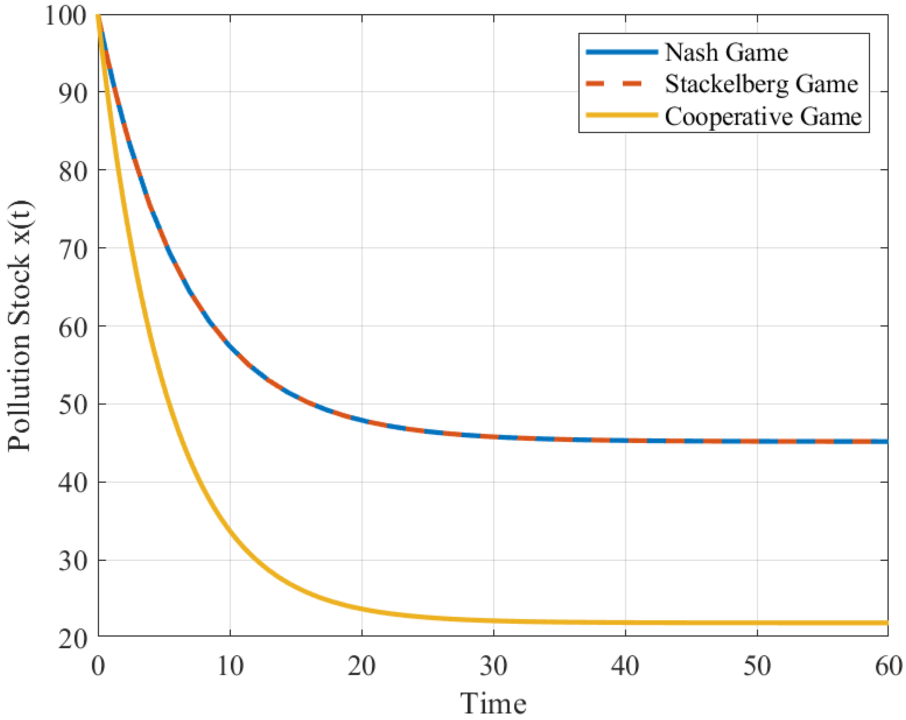

Figure 1. Dynamic Evolution of Pollution Stock.

Besides, the corporate emission reduction effort levels under different game scenarios are shown in **Figure 2.**

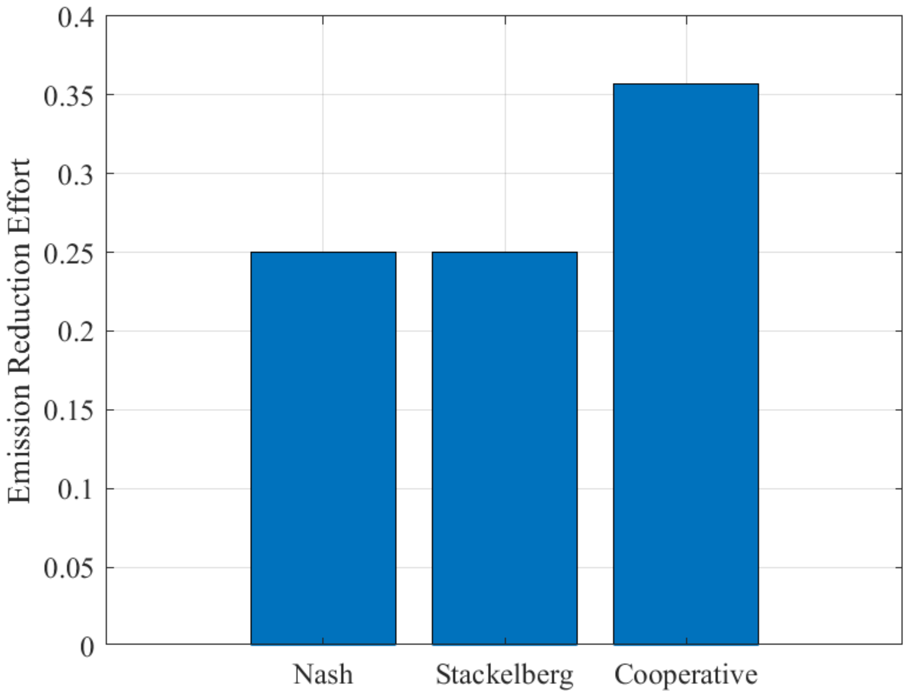

Figure 2. Enterprise Emission Reduction Effort.

Figure 2 shows that corporate emission reduction effort levels are low and nearly identical under the Nash and Stackelberg games, as firms bear costs alone and pursue profit maximization under non-cooperative conditions, lacking incentives for reduction. In contrast, under the cooperative game, government subsidies lower the marginal cost of emission reduction, leading to significantly higher effort levels, demonstrating that fiscal subsidies effectively stimulate firms' motivation. Therefore, cooperative governance is more conducive than administrative regulation alone to enhancing firms' long-term emission reduction willingness.

Furthermore, to further analyze the impact of government subsidy policy on corporate emission reduction behavior, this paper conducts a sensitivity analysis of the subsidy coefficient α, and the results are shown in **Figure 3**.

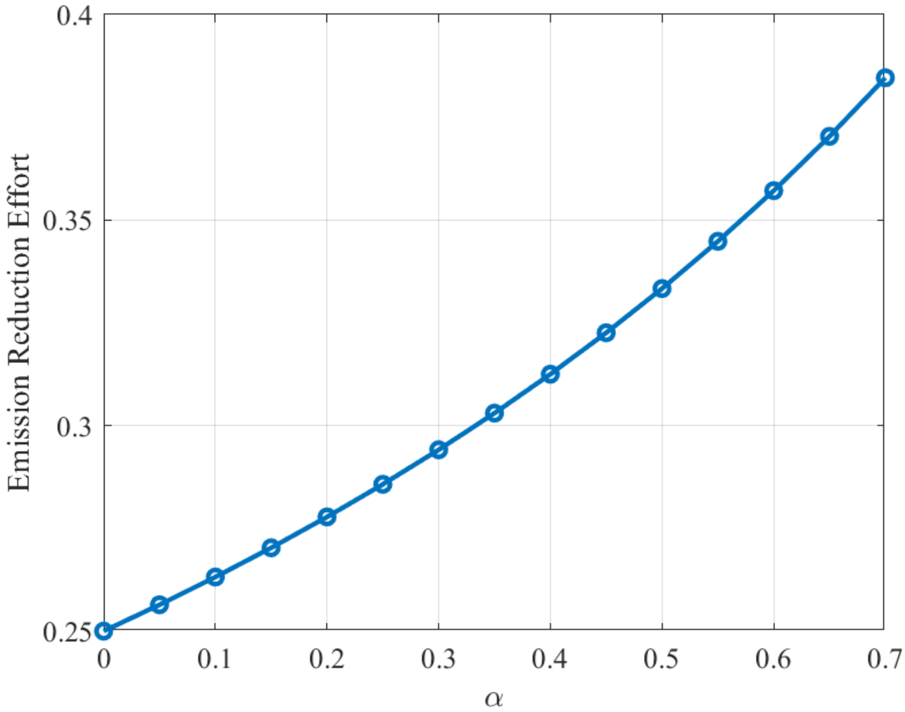

Figure 3. Sensitivity Analysis of Subsidy Coefficient.

Figure 3 shows that a higher subsidy coefficient leads to continuously rising corporate emission reduction efforts, indicating that subsidies lower abatement costs and enhance governance motivation, with an incentive amplification effect. In summary, the sensitivity analysis validates the significant advantages of the cooperative governance mechanism in improving pollution control efficiency.

Although higher subsidy coefficients continuously increase corporate emission reduction efforts, the marginal environmental benefits gradually diminish as subsidy levels rise. Excessively high subsidies may impose substantial fiscal burdens on governments, especially in regions with limited public budgets, and may also reduce overall regulatory efficiency. Therefore, a higher subsidy level is not always better. In actual environmental governance practice, governments need to set reasonable subsidy intensities based on regional economic development levels, the severity of pollution, and firms' abatement capabilities, thereby striking a balance between environmental benefits and fiscal sustainability. Moreover, excessive dependence on subsidies may weaken firms' intrinsic motivation for environmental governance and create long-term reliance on government support. Therefore, subsidy policies should be gradually integrated with technological innovation incentives and market-based environmental instruments.

6. Conclusions

This paper constructs a differential game model to analyze the dynamic interactive relationship between the government and enterprises in the process of air pollution governance. Three distinct governance scenarios are examined: Nash non-cooperative game, Stackelberg leader-follower game, and cooperative game. By comparing the dynamic evolution of pollution stock and the strategic choices of both parties under different governance mechanisms, the interactive relationship between government regulation and firms' abatement behavior is systematically analyzed. Numerical simulation results further validate the significant advantages of cooperative governance mechanisms and government subsidy policies in improving pollution control effectiveness.

We can draw the following conclusions: The government should establish a government-enterprise collaborative governance mechanism by compensating enterprises for emission reduction costs through fiscal subsidies and tax breaks, thereby stimulating their endogenous motivation for emission reduction. It is also essential to optimize the dynamic combination of subsidies and regulations by linking subsidies to emission reduction efficiency, implementing tiered and classified supervision, and strengthening the oversight of subsidy funds. Furthermore, efforts should be made to increase the research, development, and application of emission reduction and supervision technologies, as well as to build an intelligent supervision platform to enhance governance efficiency. Enterprises may be classified according to pollution intensity, environmental performance, and historical compliance records, allowing regulatory resources and subsidy policies to be allocated more efficiently. Such platforms may integrate real-time environmental monitoring systems, AI-assisted pollutant detection technologies, satellite observations, and digital reporting mechanisms to improve monitoring accuracy and reduce information asymmetry between regulators and enterprises.

The research findings also offer practical insights for air pollution control in some traditional heavy industrial regions of the United States. The U.S. Rust Belt has long been dominated by steel manufacturing, coal-based energy, and energy-intensive industries, where local enterprises often face high abatement costs and significant pressure for industrial transformation. In this context, if governance relies solely on administrative penalties and mandatory regulation, enterprises may lack sustained incentives for emission reduction due to excessive cost burdens, potentially leading to industrial relocation, job losses, and other economic problems. Therefore, these regions are better suited to adopt a collaborative governance model that emphasizes both regulation and incentives. On one hand, local governments should continue to strengthen environmental governance, raising emission standards for highly polluting enterprises and increasing the cost of non-compliance. On the other hand, financial subsidies, green tax incentives, and technology upgrade support policies are needed to help enterprises reduce the costs of green transformation, encouraging them to gradually transition toward low-carbon and cleaner production methods. Meanwhile, for regions heavily dependent on traditional energy sources, further efforts should be made to promote clean energy substitution and industrial structure upgrading. Developing new energy industries, advanced manufacturing, and green technology sectors can help alleviate the long-term environmental pressures associated with traditional heavy industries. Only by achieving a dynamic balance between environmental governance and economic transformation can the dual goals of air quality improvement and sustainable regional economic development be realized. Another challenge is the potential for industrial relocation and carbon leakage. If environmental regulations become substantially stricter in one region while neighboring regions maintain relatively lax standards, pollution-intensive industries may relocate to areas with lower compliance costs. Therefore, environmental governance should be accompanied by broader regional cooperation and coordinated regulatory frameworks to prevent pollution transfer and ensure long-term environmental effectiveness.

It should be noted that the above discussion is intended as a qualitative policy interpretation rather than a regionally calibrated empirical analysis. Compared with technology-intensive industries, traditional sectors such as steel manufacturing, coal-fired power generation, and heavy chemical production generally face higher pollution control costs and slower technological upgrading processes. Under such circumstances, subsidy-supported cooperative governance may play a more important role in encouraging firms to undertake emission reduction activities.

However, it must be pointed out that this study only considers two core governance entities—government and enterprises. Although this simplified approach helps to better focus on the direct interaction mechanisms between the government and enterprises, it may not fully capture the multi-layered interest structures and complex game relationships inherent in real-world air pollution governance. In actual governance scenarios, public environmental supervision and rights protection actions, technical services and market participation by third-party professional governance institutions, public pressure from the media and environmental NGOs, as well as coordination and cooperation among cross-regional governments, all have non-negligible impacts on pollution control outcomes. Consequently, the equilibrium outcomes derived from the current two-player framework should be interpreted with caution when applied to complex real-world environmental governance systems. For example, public reporting and environmental public interest litigation often provide effective supplements to regulatory blind spots, while market-based operations of third-party governance institutions may achieve more efficient emission reduction at lower costs. Therefore, future research should expand the analytical dimensions of the model by incorporating multiple stakeholders—such as public participation, third-party governance institutions, environmental organizations, media supervision, and horizontal collaboration mechanisms among local governments—into the modeling framework. By constructing more comprehensive multi-agent game models, researchers can more accurately depict the power structures, information asymmetries, and interest conflicts in air pollution governance, thereby enhancing the descriptive and prescriptive value of theoretical models for real-world governance practices. Recent advances in AI-assisted environmental monitoring and neural-network-based pollutant detection systems may improve information transparency and reduce regulatory costs, thereby enhancing the effectiveness of cooperative air pollution governance [26]. In addition, future research could further explore more complex combinations of policy instruments—such as subsidies, regulation, taxation, and carbon trading—that governments may adopt at different governance stages, as well as the applicability and effectiveness of these policies under different levels of economic development and industrial structures.

In addition to the stakeholder limitations discussed above, several simplifying assumptions adopted in this study may also affect the applicability of the equilibrium results in practical environmental governance settings. The equilibrium strategies derived in this study are based on the assumptions of perfect rationality and complete information. However, in practical environmental governance, governments and enterprises may exhibit bounded rationality, limited information-processing capabilities, and imperfect knowledge of pollution conditions. These factors may affect governance outcomes and the applicability of the equilibrium strategies derived in this study. Similarly, uncertainty in monitoring technologies may reduce the effectiveness of government supervision and weaken the incentive effects of regulatory policies. Future research may incorporate bounded rationality, information asymmetry, AI-assisted environmental monitoring technologies, and stochastic disturbances into differential game frameworks to further examine their impacts on governance performance and environmental quality.

Finally, the present study assumes a single regulatory authority and does not explicitly consider strategic interactions among multiple governmental entities. Future research may extend the current differential game framework by introducing multiple governmental agents with heterogeneous regulatory objectives and budget constraints. Such an extension would allow the investigation of intergovernmental strategic interactions, regional environmental cooperation, and free-rider problems arising from transboundary pollution.

Author Contributions

F.S. proposed the research direction, designed the overall research framework, and supervised the entire research and writing process. M.H. was responsible for the manuscript writing and the implementation of the study. Z.Z., Z.H., and Z.C. participated in literature review, data organization, and manuscript revision. All authors have read and approved the final version of the manuscript.

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Informed Consent Statement

Not applicable.

Data Availability Statement

The data supporting the findings of this study are available within the article. No additional datasets were generated or analyzed during the current study. MATLAB simulation codes and related materials are available from the corresponding author upon reasonable request.

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Conflicts of Interest

The authors declare no conflict of interest.

AI Use Statement

During the preparation of this manuscript, the authors used DeepSeek and ChatGPT solely for language refinement. No AI tools were used for data analysis, interpretation, or generation of scientific content. All outputs were critically reviewed and edited by the authors. The authors take full responsibility for the integrity and accuracy of the work.

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